

Introduction to (space) Plasma Physics

STFC Introductory Course in Solar and Solar-Terrestrial Physics

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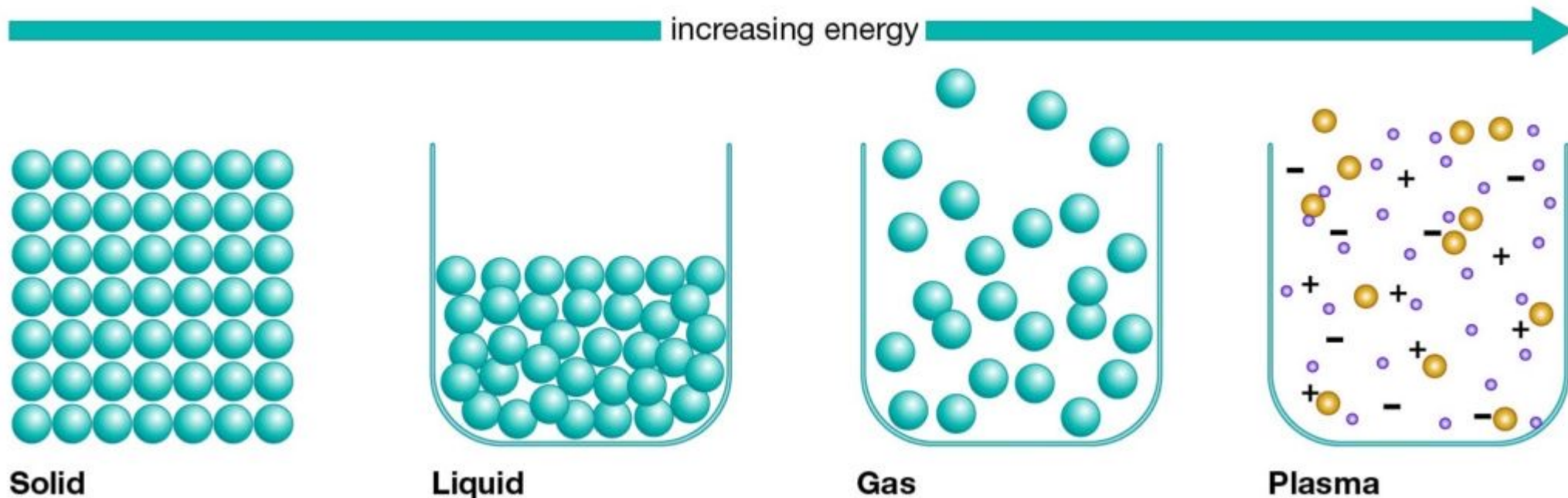
*Very much an introduction
and not self-contained
Go forth and read! 😊*

- Fundamentals: “what is a plasma?”
- What do particles and electromagnetic fields do, in a plasma?
- How do particles and fields talk to each other?
- The kinetic approximation
- The fluid approximation, including magnetohydrodynamics



What is a plasma?

[Chen, B&T,
Fitzpatrick]



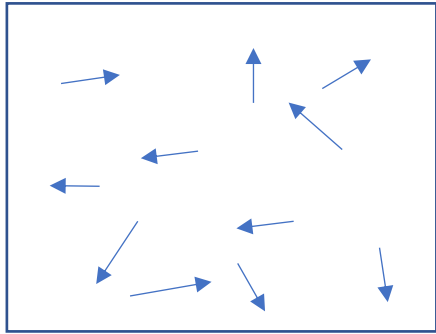
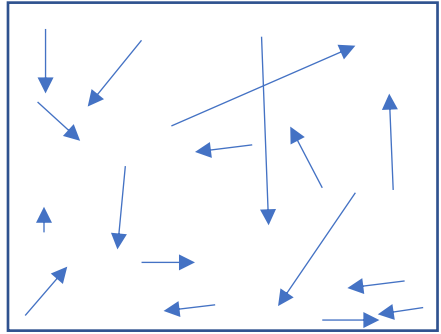
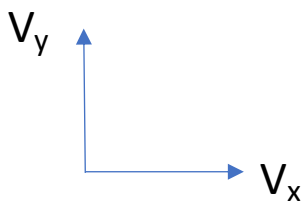
<https://studiouslyyours.com/fundamental-states-of-matter>

*“In essence **a plasma is an ionized gas...**” [Fitzpatrick]*

*“**Any ionized gas cannot be called a plasma**, of course; there is always some small degree of ionization in any gas. A useful definition is as follows: A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior.” [Chen]*

*“... **Strongly non-neutral plasmas**, which may even contain charge carriers of one sign only, occur primarily in laboratory experiments, and are **not discussed** in this book. (Interested readers are referred to Davidson 2001.)” [Fitzpatrick]*

What is temperature (I/II)?



$$f(u) = A \exp(-\frac{1}{2}mu^2 / KT)$$

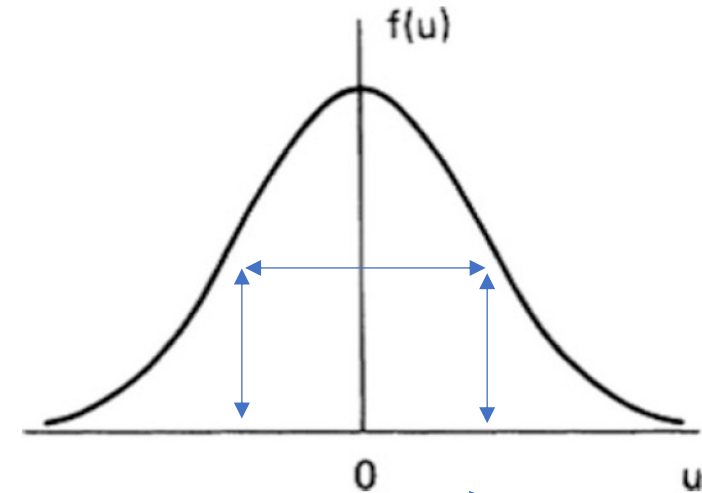
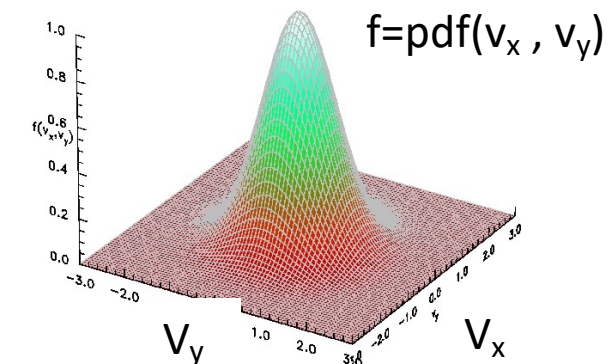
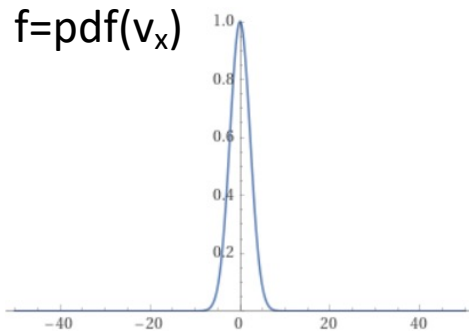
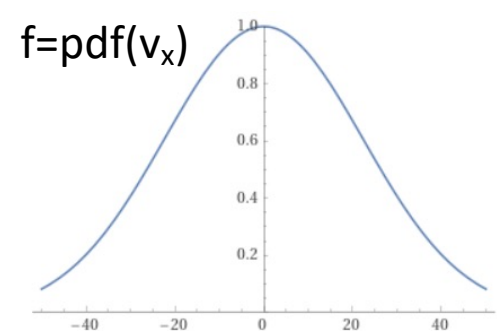


Fig. 1.2 A Maxwellian velocity distribution

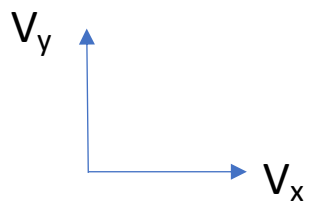


Temperature \sim random motion (velocities) of particles
Temperature \propto mass * variance of the velocities

Thermal velocity \propto standard deviation i.e. $v_{th} \propto \sqrt{T}$

Temperature \rightarrow Average kinetic energy ...
... but not necessarily the other way round

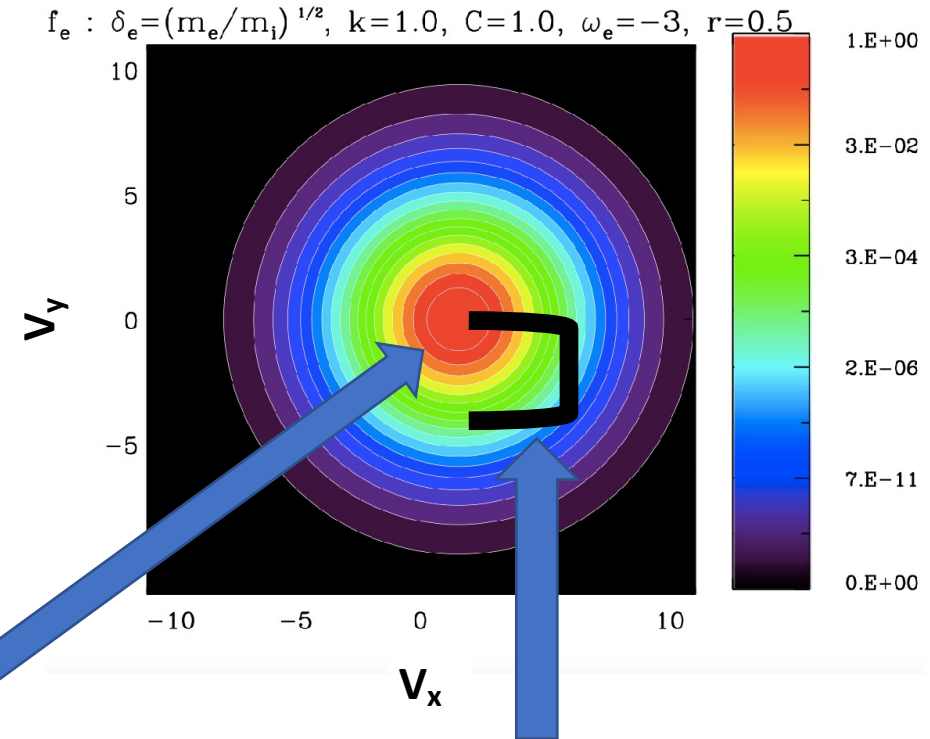
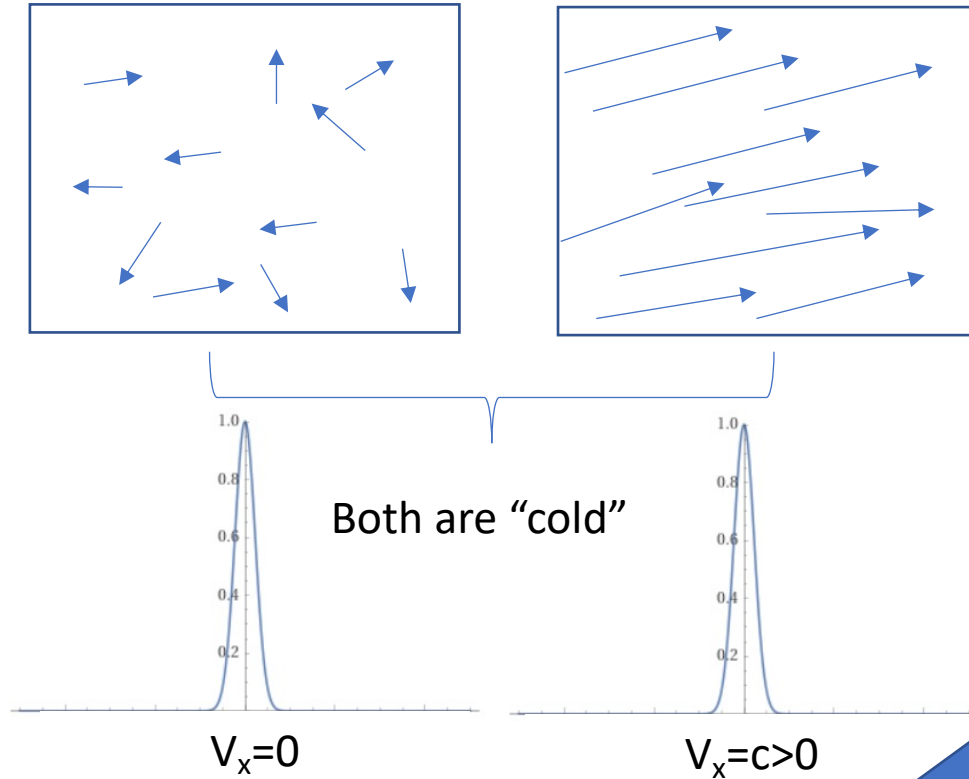
What is temperature (II/II)?



The variance is independent of the mean!

-Temperature is in principle directional as T_{xx} , T_{yy} , T_{zz} , could all be different. It may not even be defined! (e.g. see B&T ch. 6)

-Heat is the transfer of thermal energy from one system to another due to a temperature difference



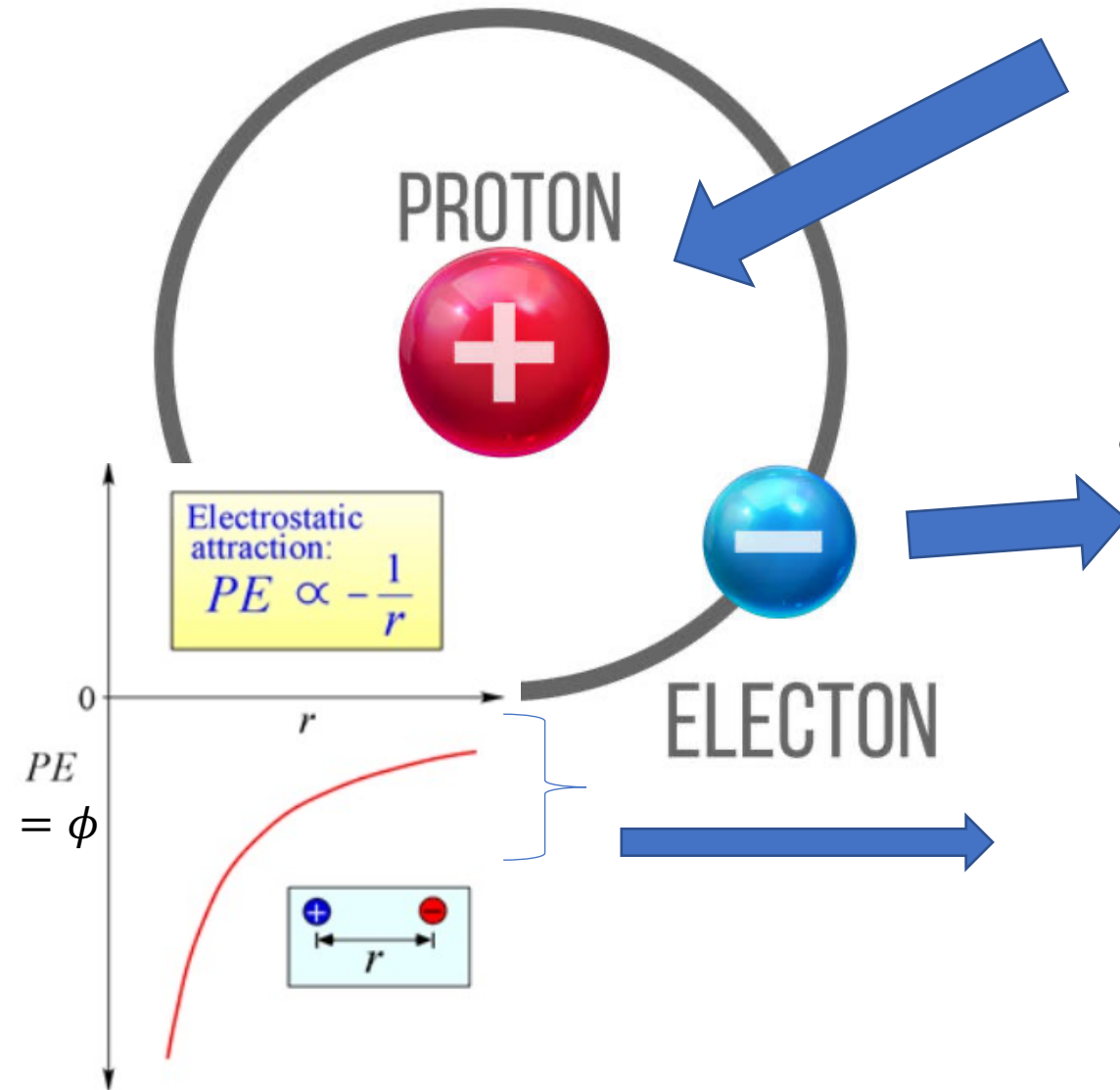
The bulk flow \mathbf{V}_s

\sim Temperature $k_b T_s = m_s v_{th,s}^2$

$$f_s(\mathbf{x}, \mathbf{v}, t) = \frac{n_s(\mathbf{x}, t)}{(\sqrt{2\pi} v_{th,s})^3} e^{-(\mathbf{v} - \mathbf{V}_s(\mathbf{x}, t))^2 / (2v_{th,s}^2)}$$

How do I make a plasma? ... ionise!

HYDROGEN



The electron is stuck in a potential well with potential energy

$$\phi = \text{potential energy} = q\varphi$$

$$\varphi = \text{electrostatic potential}$$

$$(E = -\nabla\varphi \text{ electrostatic})$$

$$\mathbf{F} = q\mathbf{E} = -q\nabla\varphi = -\nabla\phi, \text{ for Coulomb force only (conservative)}$$

$$|\mathbf{F}| = q_e^2 / (4\pi\epsilon_0 r^2) \sim O(10^{39}) * \text{gravity}$$

$$\text{acceleration} \sim |\mathbf{F}|/m \sim 10^{22} \text{ times stronger than at surface of Earth}$$

$$\text{A typical electron is characterised by some kinetic energy} \sim K_B T \sim (1/2) m v_{th}^2$$

Electron can be freed if $KE > PE$, i.e. $\sim K_B T > |\phi|$

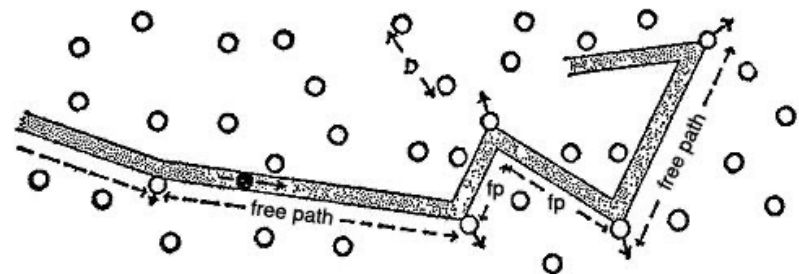
This typically requires an energy of a few electron volts (eV)

Hydrogen ionisation energy is 13.6 eV

1 eV (energy) \sim 11,000 Kelvin (temperature)

We often quote temperature "in eV", ok since $temp = energy/K_B$

Particle collisions

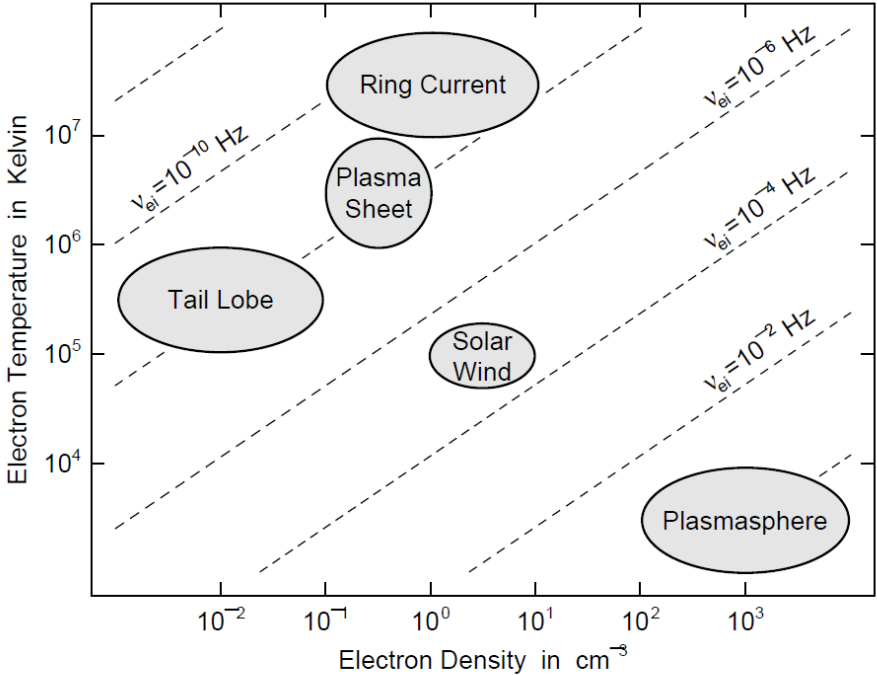


Mean free path of a gas molecule

Table 1: Varying Coulomb collisions

Parameter	Chromosphere (1.01 R_{\odot})	Corona (1.3 R_{\odot})	Solar wind (1 AU)
$n(\text{cm}^{-3})$	10^{10}	10^7	10
$T(\text{K})$	10^3	$1 - 2 \times 10^6$	10^5
$\lambda_c(\text{km})$	1	10^3	10^7

Living Reviews in Solar Physics
<http://www.livingreviews.org/lrsp-2006-1>



Baumjohann & Treumann

Fig. 4.2. Typical Coulomb collision frequencies for geophysical plasmas.

$Collision\ freq \sim n^{1/2} * \ln (T^{3/2}/n^{1/2}) / (T^{3/2} / n^{1/2})$

High n , low T

Low n , high T

“Too many collisions” --- a non-negligible amount of collisions --- “very rare collisions”

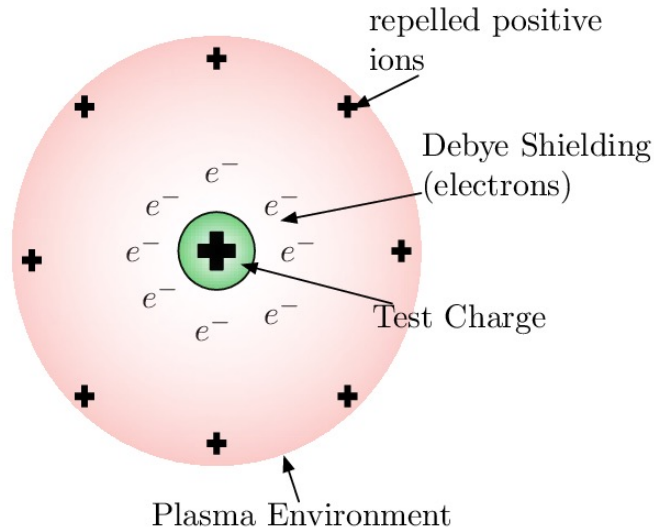
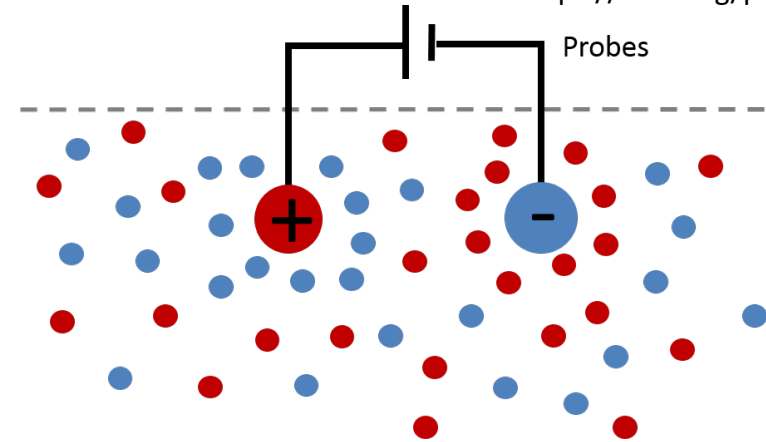
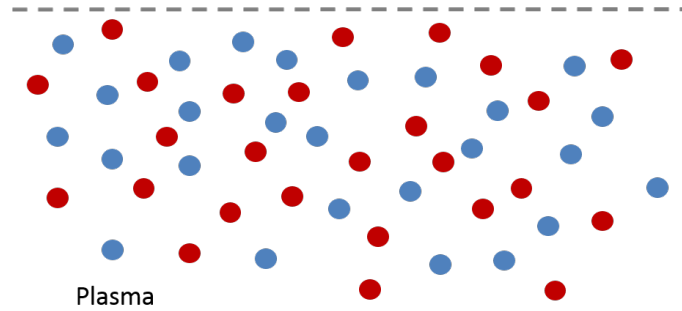
Not a plasma --- collisional plasma --- Collisionless plasma

Recombination (Saha equation) --- well defined temperature --- temperature may be tricky to define

“If you remember nothing else about plasmas”: 1

Debye sphere & quasineutrality

<https://arxiv.org/pdf/1705.10529.pdf>



$$\lambda_D \equiv \left(\frac{\epsilon_0 K T_e}{n e^2} \right)^{1/2}$$

Non-neutrality / significant electric potentials on scales below “Debye length”
Debye radius determined by point where thermal energy ($\sim K_B T$) matches electric potential ($\sim 1/r^2$)
Essentially a “statistical version of the ionisation slide”

Scales greater than λ_D , plasma appears neutral, with $n_i \approx n_e$
 If length scales $L \gg \lambda_D$, then we have “quasineutrality” and the bulk of the plasma is free of large electric fields
(May be challenged in non-thermal and very hot plasmas)

“If you remember nothing else about plasmas”: 2

Langmuir waves (plasma oscillations)

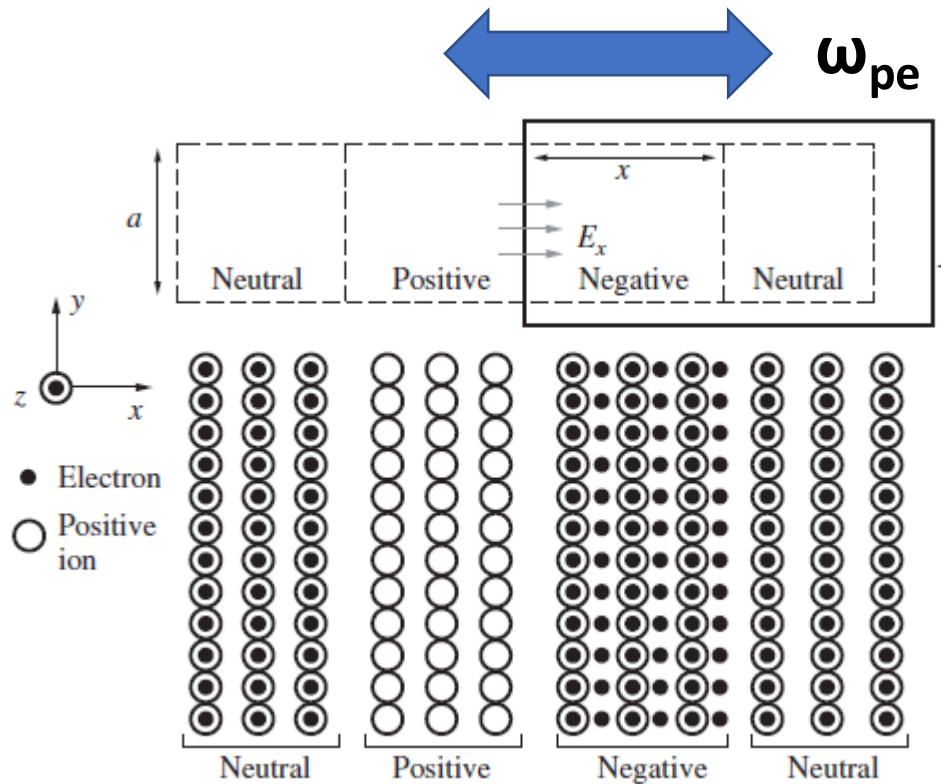


Image source: *Principles of Plasma Physics*, Inan and Golkowski

Quoting Freidberg (1987) directly: “For any low-frequency macroscopic charge separation that tends to develop, the electrons have more than an adequate time to respond (i.e. of the order $1/\omega_{pe}$), thus creating an electric field which maintains the plasma in local quasineutrality”.

$$\omega_{pe} = \left(\frac{n_e e^2}{m_e \epsilon_0} \right)^{1/2}$$

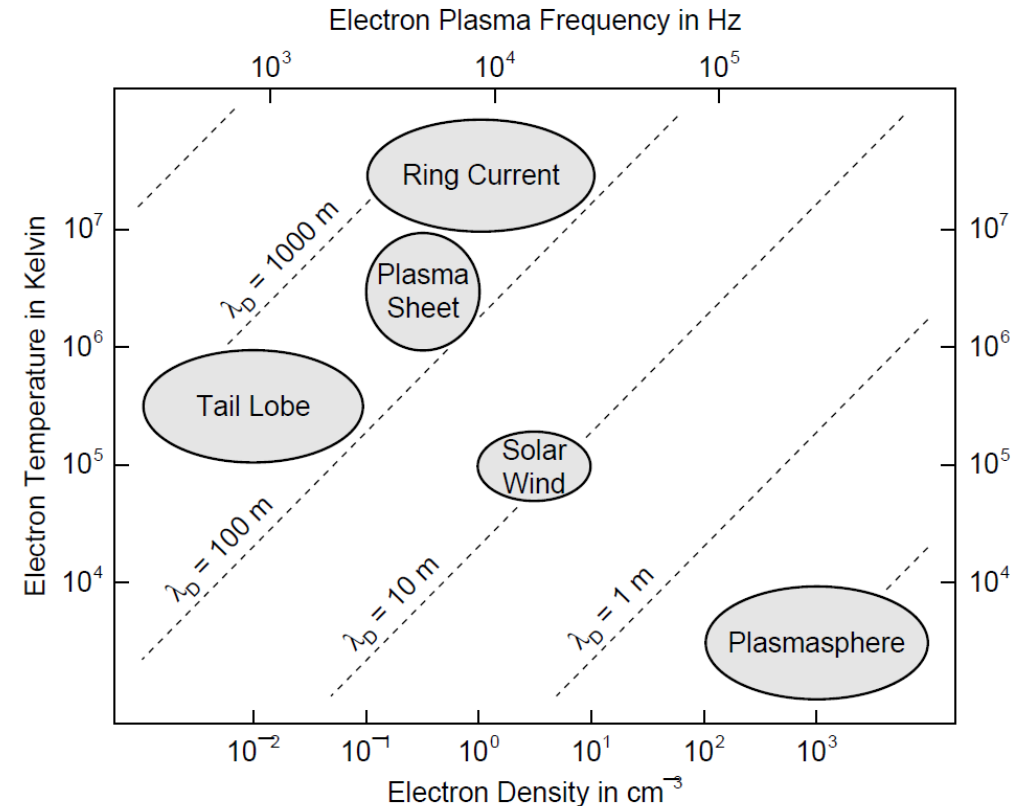
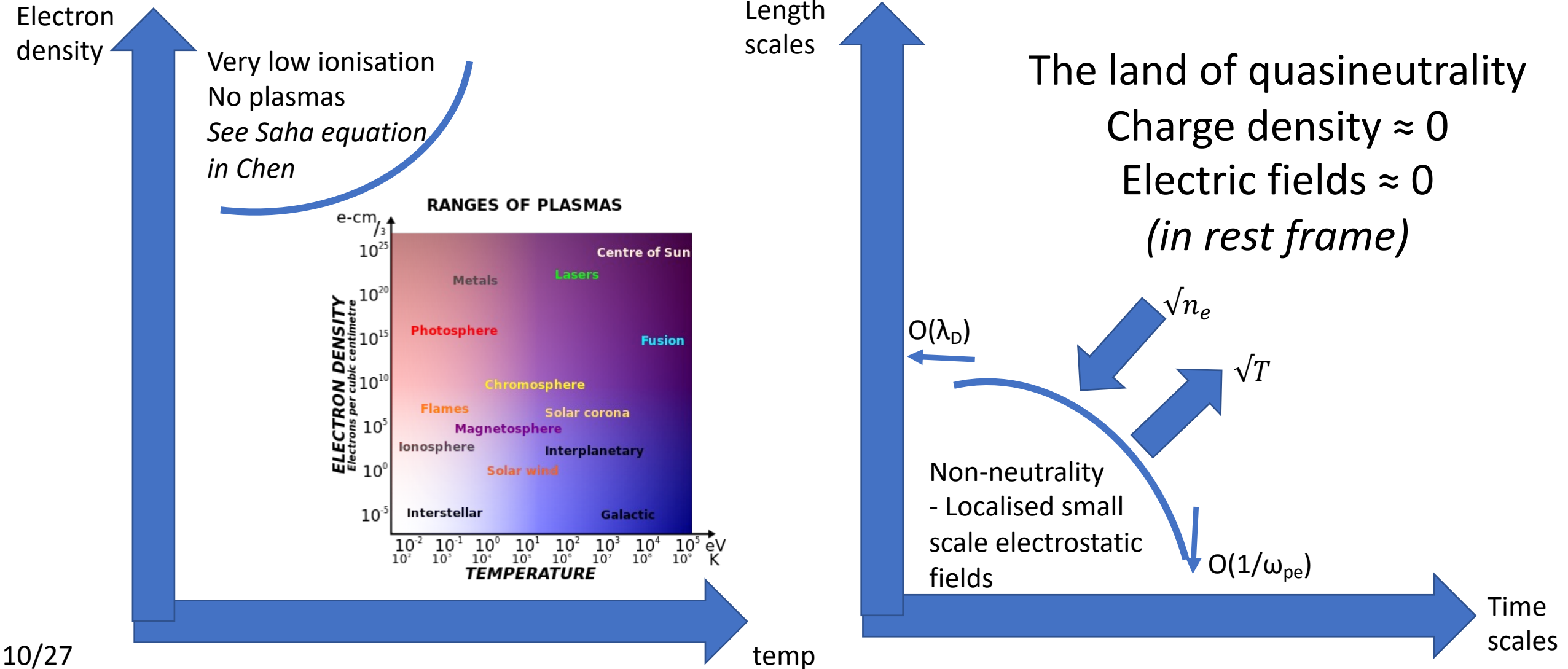


Fig. 1.2. Ranges of typical parameters for several geophysical plasmas.

Summary

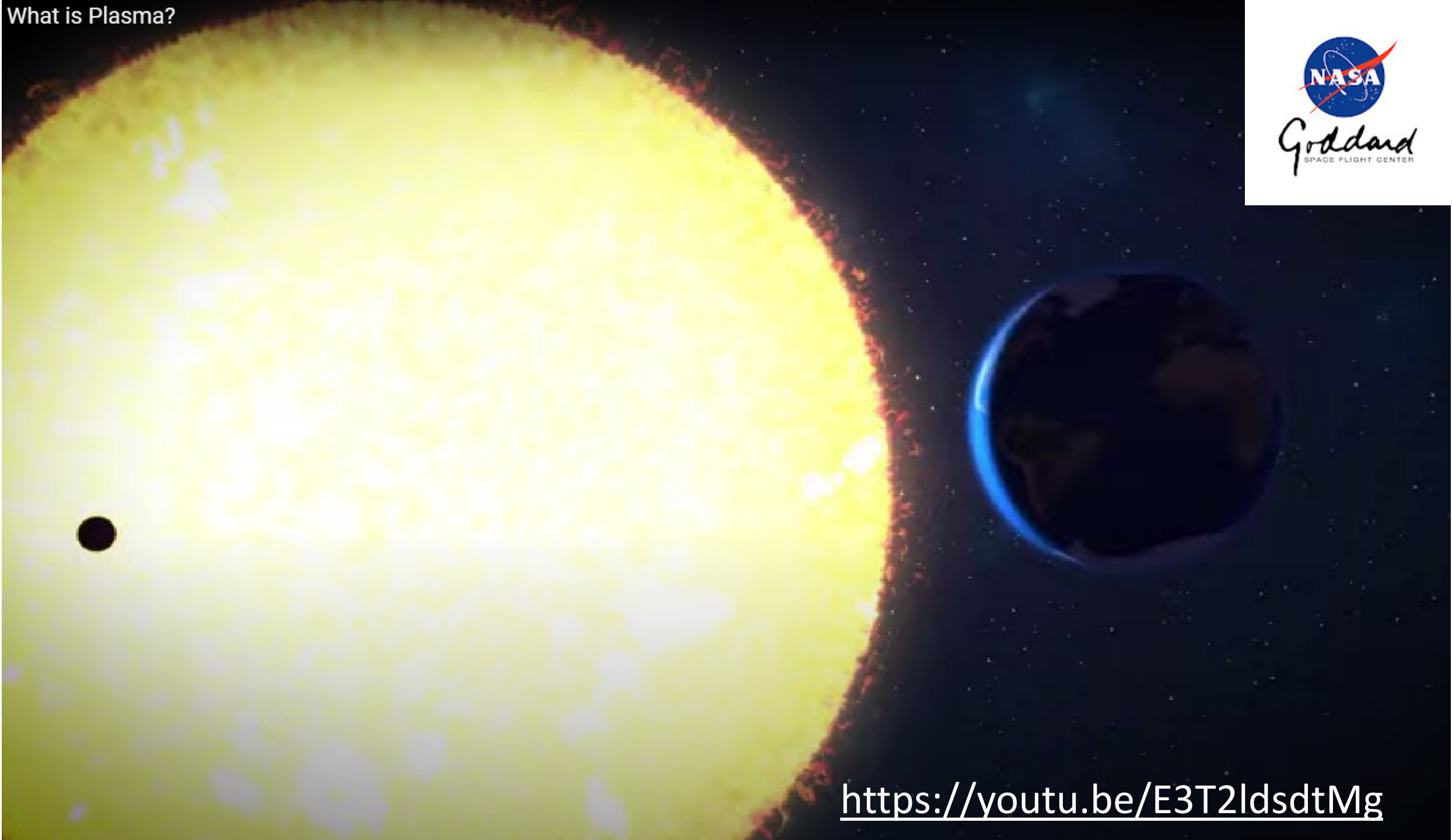


Why do plasmas exist? ✓

What has it got to do with space science?
Isn't space cold and empty?

More than 95/99/99.9% of the
“normal matter” is in the plasma state
of matter
[all plasma textbooks]

What is Plasma?



<https://youtu.be/E3T2ldsdtMg>

BREAK

- We know “what is plasma?” and “why?”
- Next is “how”!
- How will we describe particles and electromagnetic fields physically/mathematically?

Maxwell's Equations

$$\mathbf{E} = -\nabla\varphi - \frac{\partial\mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

φ electrostatic potential
 \mathbf{A} magnetic vector potential

Electric
field

Magnetic
field

Faraday's Law

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t,$$

Time-varying magnetic fields create electric fields

Ampere's Law
(with displacement current)

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t},$$

Currents (aka flowing charges) create magnetic fields and fast fluctuations in the electric field

Gauss' Law

$$\nabla \cdot \mathbf{E} = \sigma / \epsilon_0,$$

Charges create electric fields

Solenoidal constraint

$$\nabla \cdot \mathbf{B} = 0.$$

Current
density

Charge
density

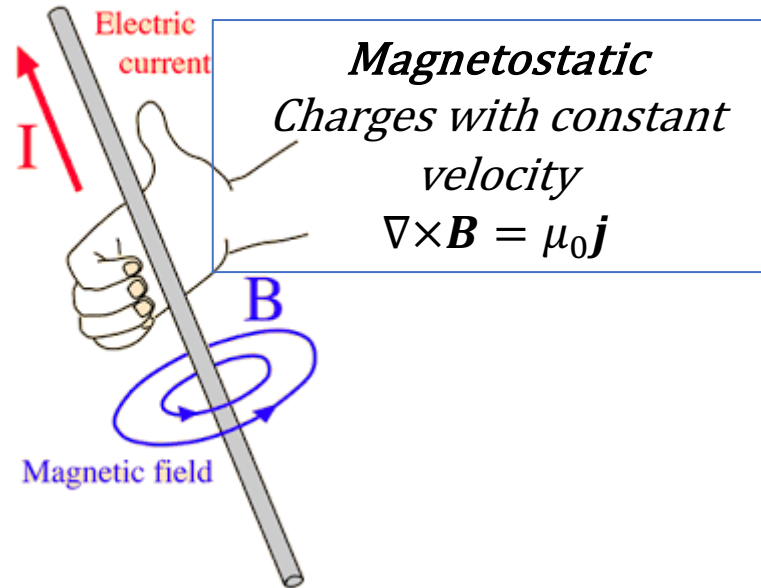
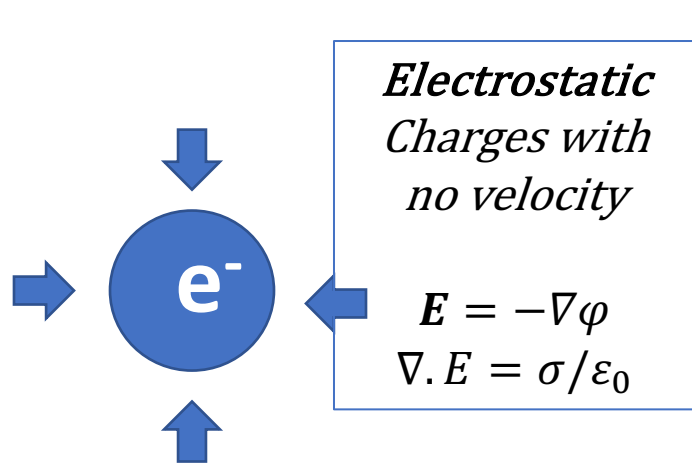
There are no "sources" of the magnetic field, and magnetic field lines must close

Charge and current densities are known as the "source terms"

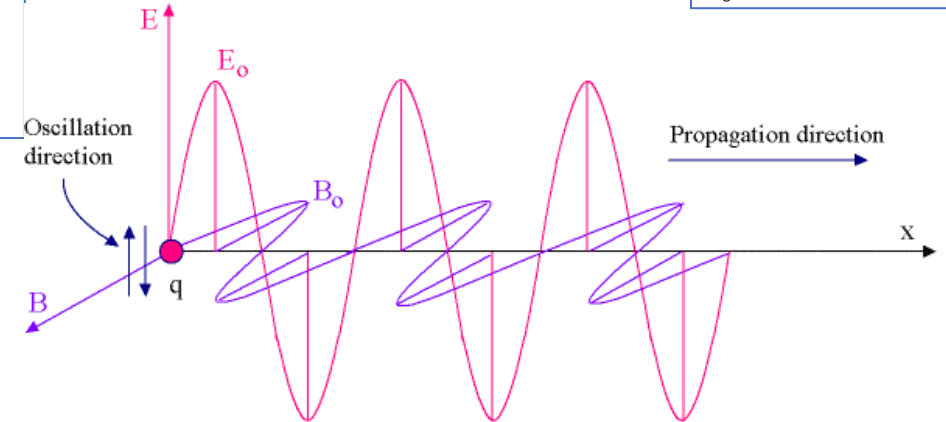
$$\sigma = \sum_s q_s n_s \quad \mathbf{J} = \sum_s q_s n_s \mathbf{v}_s$$

EM waves (in a vacuum)

[Griffiths, Jackson]



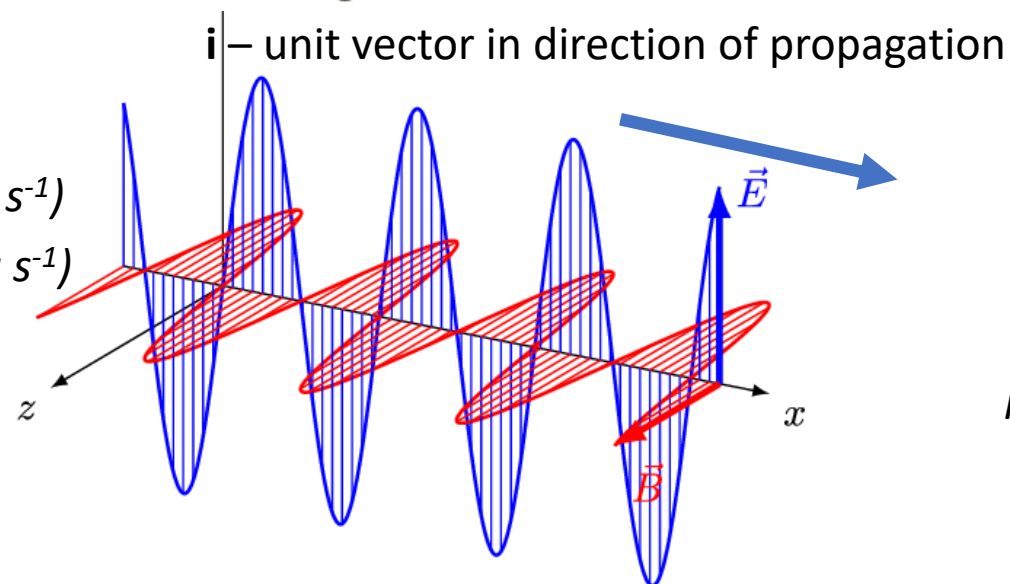
<p>Electrodynamics Accelerating charges Electromagnetic wave</p>	$\frac{1}{c_0^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0$ $\frac{1}{c_0^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$
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period of oscillation = T (s)
ordinary frequency $f = \frac{1}{T}$ (Hz aka s^{-1})
angular frequency $\omega = 2\pi f$ (Hz aka s^{-1})

wavelength = λ (m)
wavenumber $k = 2\pi/\lambda$ (m^{-1})

wave vector $\mathbf{k} = k \mathbf{i}$



let f , i.e. ω , define a wave mode in a vacuum $\lambda = f/c$, i.e. $\omega = ck$

This is the “dispersion relation”
 $\omega = \omega(k)$ in general

Mode travels at **phase velocity** $v_{ph} = \omega/k$

Refractive index $n = kc/\omega$
 $|n| = c/v_{ph} = 1$ in a vacuum

Plasma waves - Dielectric tensor

[B+T, T+B, Stix,
G&B, Freidberg]

A general philosophy for deriving wave modes in “many” situations:

Assume an infinite uniform plasma constant background \mathbf{B} , i.e with $\mathbf{E}_0 = \mathbf{0}$ and e.g. $\mathbf{B}_0 = B_0 \hat{z}$

Start by assuming microscopic Maxwell equations ie vacuum permittivity and permeability, $\epsilon_0 \mu_0$

Consider the influence of a plasma “immersed in a vacuum”

The dielectric tensor then tries to describe the effects of the plasma on wave propagation

Different plasmas will allow different waves

$$\begin{aligned}\nabla \times \mathbf{E} &= -\partial \mathbf{B} / \partial t, \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}, \\ \nabla \cdot \mathbf{E} &= \sigma / \epsilon_0, \\ \nabla \cdot \mathbf{B} &= 0.\end{aligned}$$

Dielectric
tensor

$$\vec{\mathbf{K}} = \vec{\mathbf{I}} + \frac{i}{\epsilon_0 \omega} \vec{\sigma}$$

$$\mathbf{J}_1 = \vec{\sigma} \cdot \mathbf{E}_1$$

Conductivity
tensor

$$\begin{aligned}i\mathbf{k} \times \mathbf{E}_1 &= i\omega \mathbf{B}_1, \\ i\mathbf{k} \times \mathbf{B}_1 &= \mu_0 \mathbf{J}_1 - \frac{i\omega}{c^2} \mathbf{E}_1, \\ i\mathbf{k} \cdot \mathbf{E}_1 &= \frac{\sigma_1}{\epsilon_0}, \\ i\mathbf{k} \cdot \mathbf{B}_1 &= 0.\end{aligned}$$

$$\mathbf{n} \times \mathbf{n} \times \mathbf{E}_1 + \vec{\mathbf{K}} \cdot \mathbf{E}_1 = 0$$

\mathbf{n} = refractive index
Dispersion relation

$$D(\omega, \mathbf{k}) = 0$$

$$\begin{vmatrix} n_y^2 + n_z^2 - K_{xx} & -n_x n_y - K_{xy} & -n_x n_z - K_{xz} \\ -n_x n_y - K_{yx} & n_x^2 + n_z^2 - K_{yy} & -n_y n_z - K_{yz} \\ -n_x n_z - K_{zx} & -n_y n_z - K_{zy} & n_x^2 + n_y^2 - K_{zz} \end{vmatrix} = 0$$

$$\mathbf{B}_1(\mathbf{r}, t) \propto \int \mathbf{B}_{\mathbf{k}} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} d^3 k d\omega$$

$$\mathbf{E}_1(\mathbf{r}, t) \propto \int \mathbf{E}_{\mathbf{k}} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} d^3 k d\omega$$

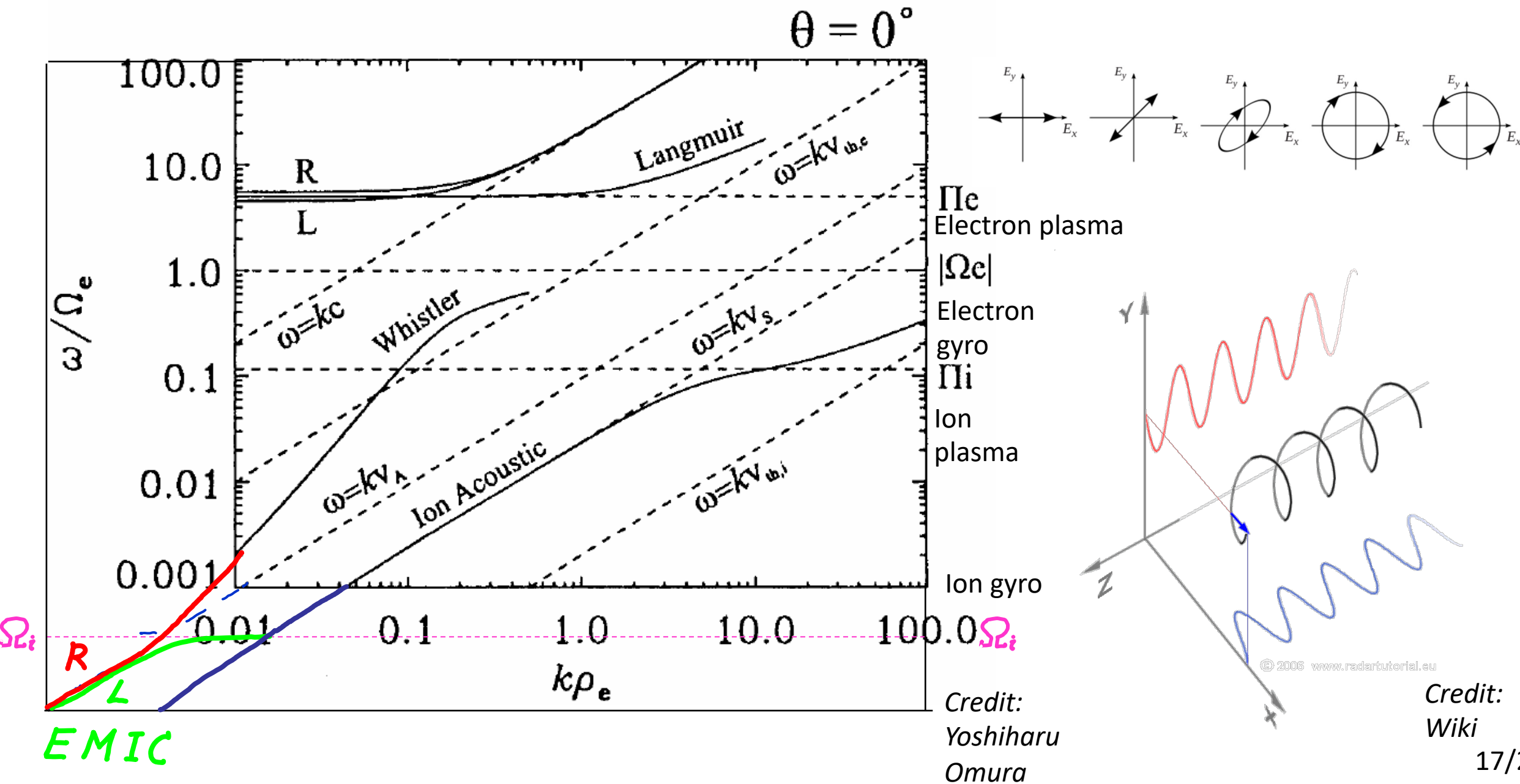
Linearisation
Fourier modes

Info on
Density,
temperature
Strength of B_0 ,
species,
ionisation...

→ Conductivity
& dielectric



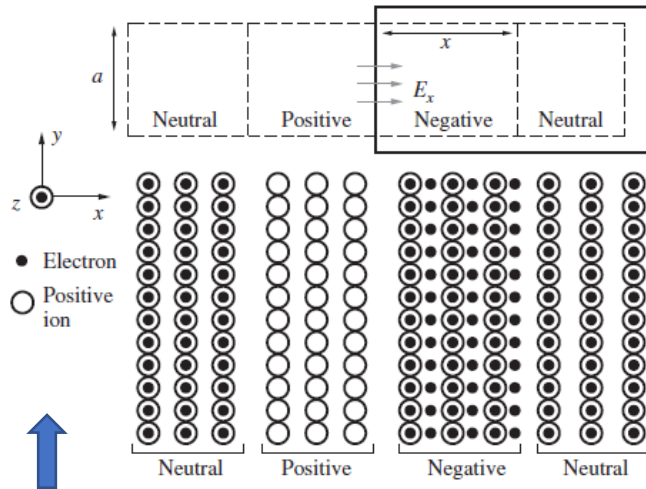
Example “Dispersion relation” (Parallel Propagation $\mathbf{k} \cdot \mathbf{B}_0 = 0$)



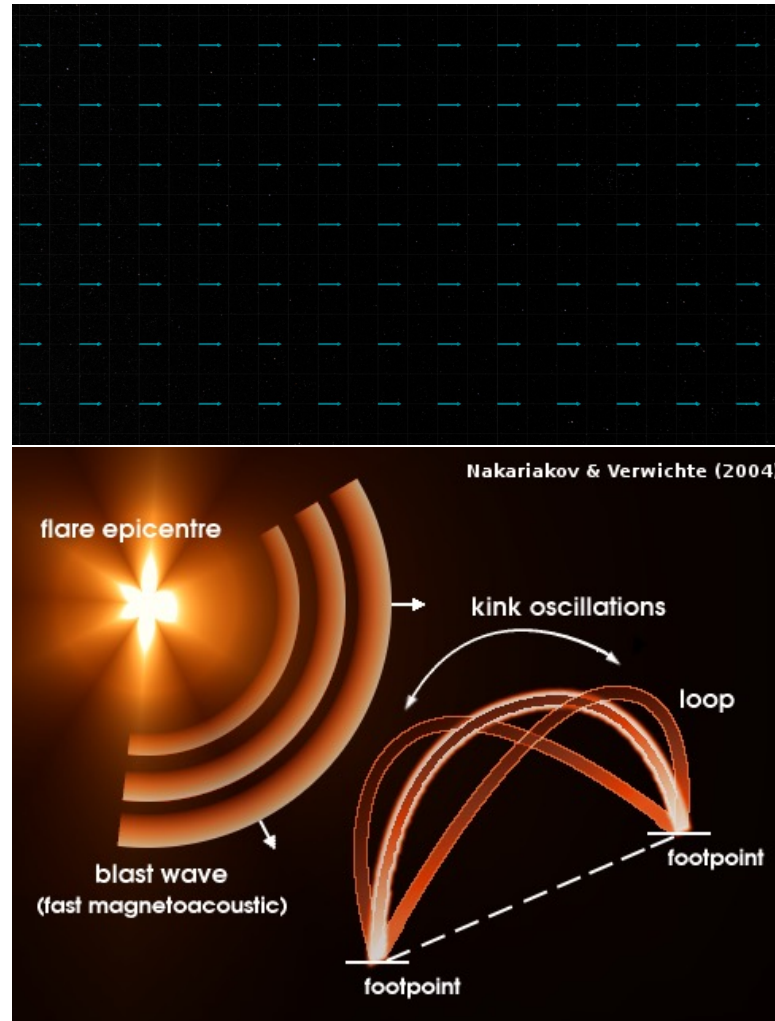
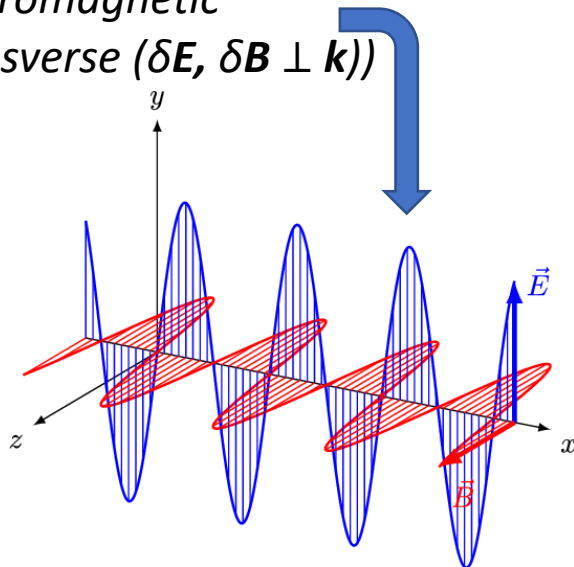
There are so so many waves

$\sim \text{kHz } (f < f_{ce})$

whistler waves (kinetic)



Electrostatic (Longitudinal ($\delta \mathbf{E} \parallel \mathbf{k}$)) vs
electromagnetic
(transverse ($\delta \mathbf{E}, \delta \mathbf{B} \perp \mathbf{k}$))



*In principle many wave modes
can be derived from a suitable
dielectric approach*

But not always necessary

*Take your given system and do
Fourier things!*

$$Q(\mathbf{r}, t) = Q_0 + \tilde{Q}_1(\mathbf{r}, t)$$

$$\tilde{Q}_1(\mathbf{r}, t) = Q_1 \exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r})$$

$f > f_{ci}$ e.g. MHD waves (fluid)

Lorentz force – particle motion

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(\gamma m_0 \mathbf{v})}{dt} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

m_0 rest mass ; $\gamma = 1/(1 - v^2/c^2)$ relativistic gamma ; q the signed particle charge

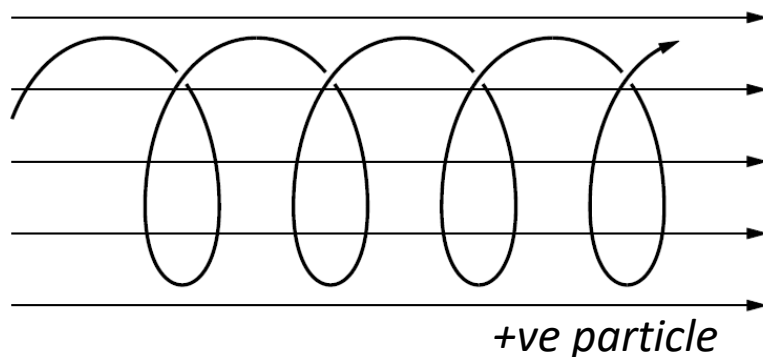
Constant magnetic field

$$\mathbf{E} = \mathbf{0}, \text{ but } \mathbf{B} = (0, 0, B_0)$$

$$\mathbf{F} = q (\mathbf{v} \times \mathbf{B})$$

$$dW/dt = \mathbf{F} \cdot d\mathbf{v}/dt = 0$$

Magnetic fields do no work i.e.
cannot change energy of a particle

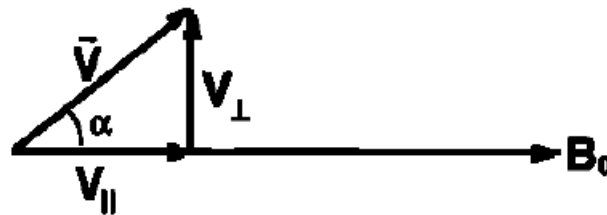


Gyro-/Cyclotron-/Larmor-frequency

$$\omega_c = q |\mathbf{B}| / (m_0 \gamma)$$

positive and negative particles gyrate in different directions

Gyro-/Cyclotron-/Larmor-radius $r_L = v_{\perp} / |\omega_c|$



$$\sin \alpha = \frac{v_{\perp}}{v}$$

α = "pitch-angle"

Constant electric field

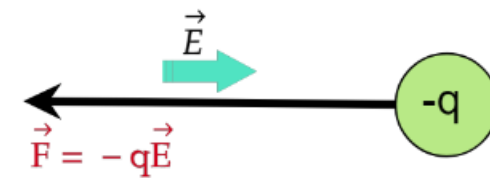
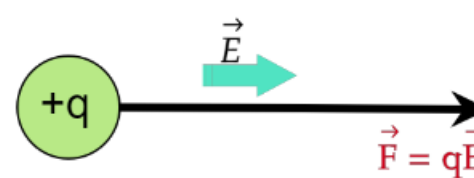
$$\mathbf{B} = \mathbf{0}, \text{ but } \mathbf{E} = (0, 0, E_0)$$

$$\mathbf{F} = q\mathbf{E}$$

$$dW/dt = \mathbf{F} \cdot d\mathbf{v}/dt = qE_0 / (\gamma m_0) \text{ - rel correction}$$

A constant electric field accelerates particles
indefinitely

It will almost but never quite reach c



Non-uniform B, electric fields, external forces (e.g. gravity)

Northrop guiding centre theory

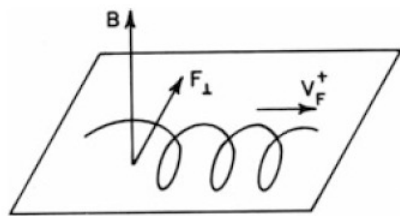


Fig. 1.6 Drift V_F^+ in a homogenous magnetic field B under a force F_{\perp}

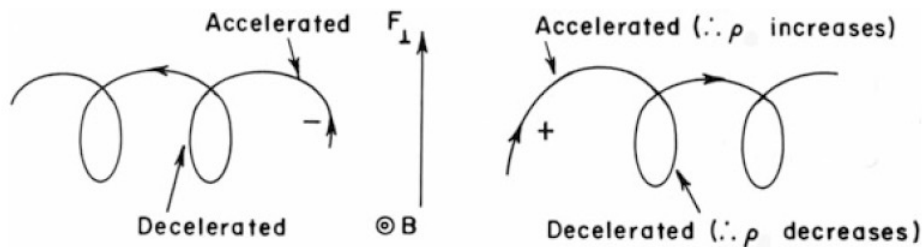
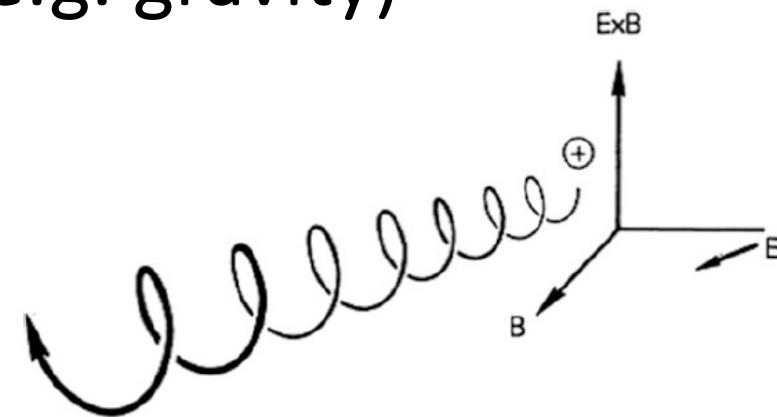
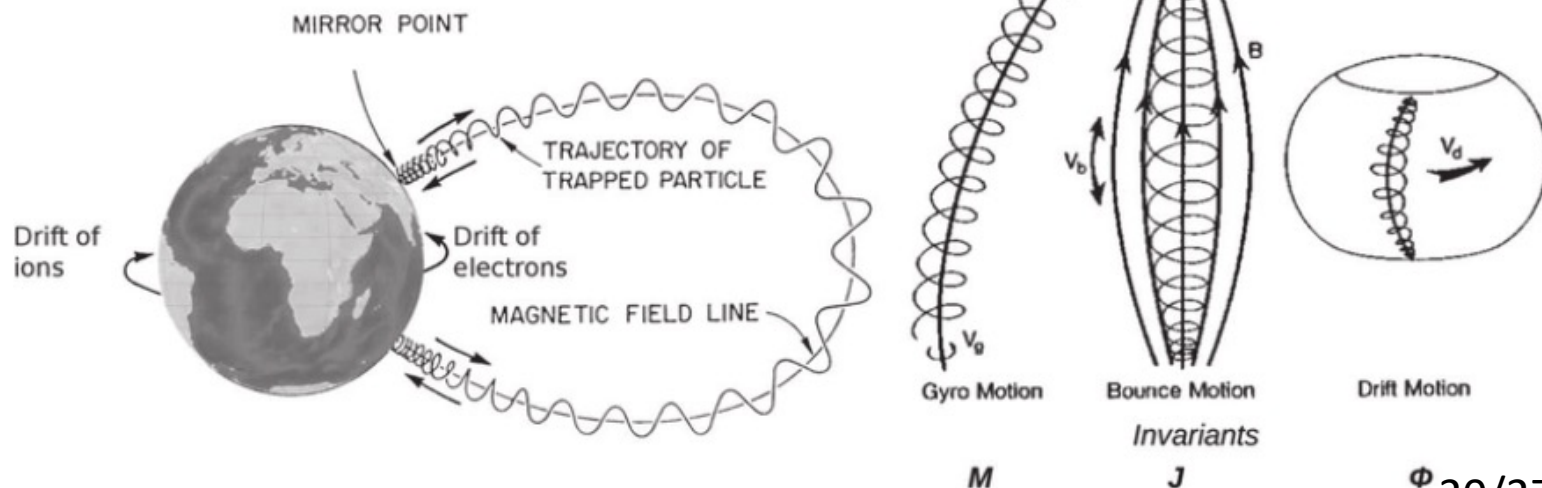


Fig. 1.7 Physical cause for the existence of a force drift

$$\mathbf{v}_f = \frac{1}{q} \frac{\mathbf{F} \times \mathbf{B}}{B^2}$$

If Larmor radius small compared to system gradients then can superpose drifts on to normal gyromotion.

If Larmor radius large, then ... tricky ... field line curvature scattering and all sorts. Not always a "recipe".

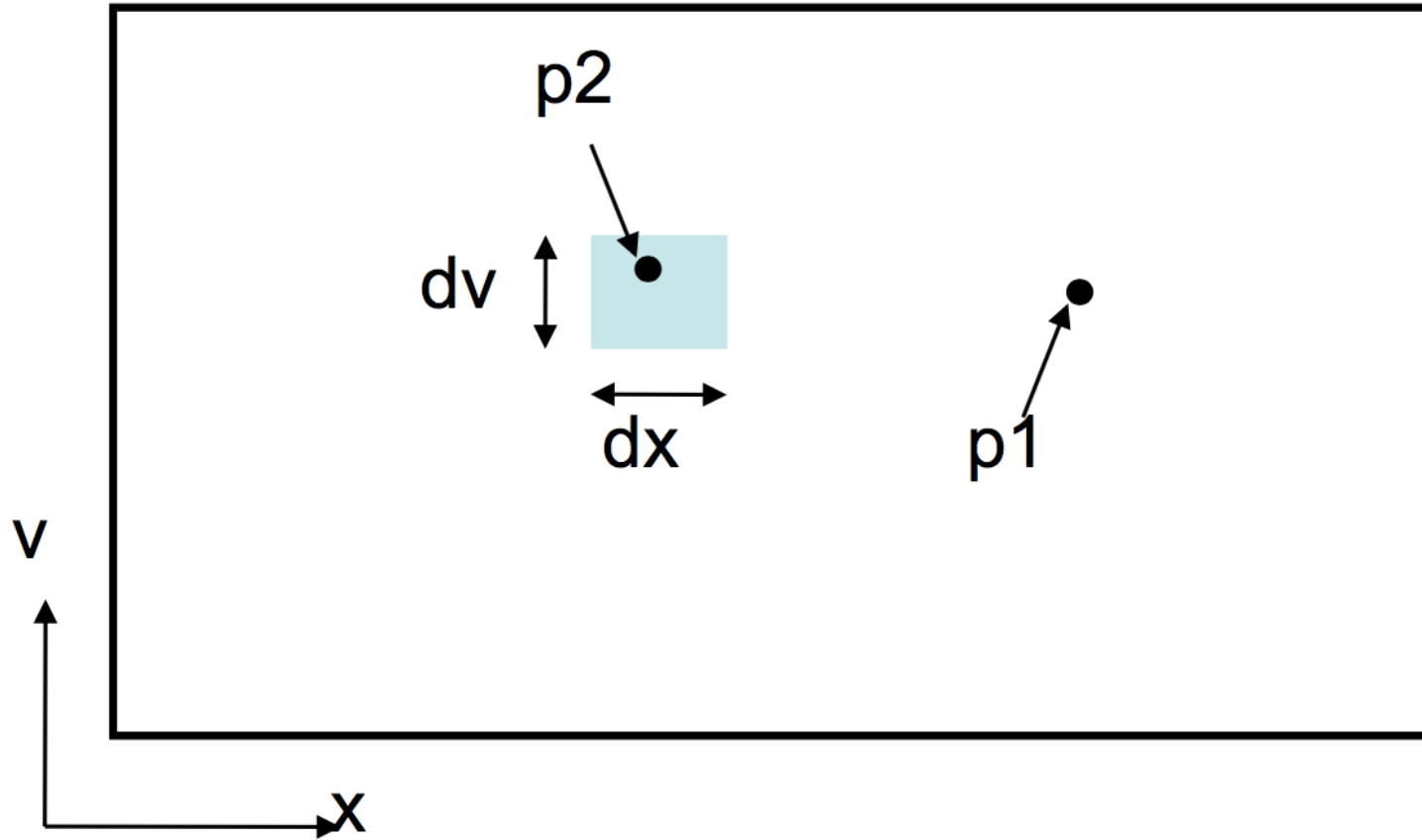


BREAK

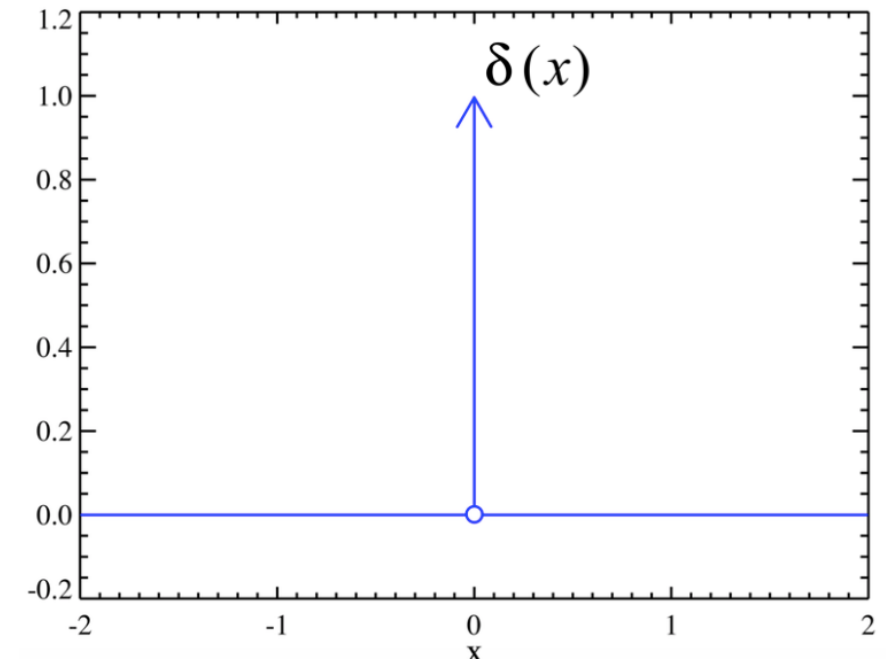
- We now know how to describe electromagnetic fields and waves
- And also the motion of individual particles
- How does everything evolve together?

Statistical description of plasma

(I): Klimontovich-Dupree



- “Pathologically jagged”
- Actually don’t care about individual particles



$$f_n(\mathbf{x}, \mathbf{v}, t) = \sum_i^N \delta(\mathbf{x} - \mathbf{x}_i) \delta(\mathbf{v} - \mathbf{v}_i)$$

(credit for both pics: Ben McMillan, Warwick)

Statistical description of plasma: (II) Distribution function

“1-particle” distribution function $f_s(\mathbf{x}, \mathbf{v}, t)$



$f_s(\mathbf{x}, \mathbf{v}; t) d^3x d^3v$ = # of particles in volume d^3x centred on \mathbf{x} with velocities in range $\mathbf{v} + d\mathbf{v}$.

Particle number density $n_s(\mathbf{x}, t) = \int f_s d^3v$

Bulk flow velocity $\mathbf{V}_s = n_s^{-1} \int \mathbf{v} f_s d^3v$

Pressure tensor $P_{ij} = \sum_s m_s \int w_{is} w_{js} f_s d^3v$

$$\sigma = \sum_s q_s n_s$$

$$\mathbf{j} = \sum_s q_s n_s \mathbf{V}_s$$

• Self-consistent electromagnetic fields



\mathbf{E}, \mathbf{B}



Statistical description of plasma (“Kinetic physics”)

(III): Boltzmann/Vlasov equation

General kinetic/Boltzmann equation

$$\frac{df(\mathbf{p}, \mathbf{r}, t)}{dt} = \frac{\partial f}{\partial t} + \frac{d\mathbf{p}}{dt} \cdot \frac{\partial f}{\partial \mathbf{p}} + \frac{d\mathbf{r}}{dt} \cdot \frac{\partial f}{\partial \mathbf{r}} = 0$$

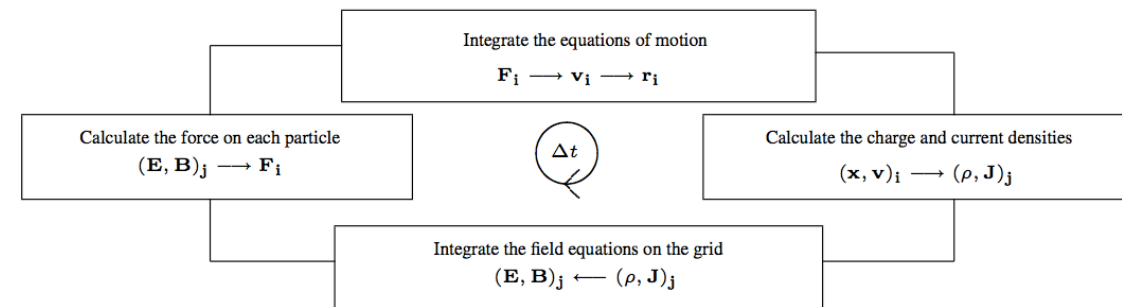
$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
 $\frac{d\mathbf{r}}{dt} = \mathbf{v}$

Collisional plasmas - Particles follow their orbits as defined by interactions with electromagnetic fields unless they collide

Collisionless plasmas - If they don't collide they follow their orbits – continuity in 6d phase space (\mathbf{p}, \mathbf{r})

$$\frac{\partial(\sqrt{g}f)}{\partial t} + \nabla \cdot (\sqrt{g}\dot{\mathbf{x}}f) = 0$$

Vlasov and particle-in-cell kinetic “self-consistent” codes for f, E, B ”



Mike Harrison PhD thesis,
St Andrews 2009


Figure 4.2: Time chart showing the general cycle of a PIC code

Fluid theories (*position space only*)

$$\frac{df(\mathbf{p}, \mathbf{r}, t)}{dt} = \frac{\partial f}{\partial t} + \frac{d\mathbf{p}}{dt} \cdot \frac{\partial f}{\partial \mathbf{p}} + \frac{d\mathbf{r}}{dt} \cdot \frac{\partial f}{\partial \mathbf{r}} = \left(\frac{\partial f}{\partial t} \right)_c$$

“nth order moment of A” = $\int A \mathbf{v}^n d^3v$

0th order moment 

1st order moment 

2nd order moment 

.... ∞

p scalar pressure
 π_{ij} stress tensor

q_{ij} heat flux
 tensor

\mathbf{F} collisional
 friction vector

W collision
 energy exchange

- Appropriate on “longer” timescales and “larger” length scales than those of the particles
- Usually characterised by thermal distributions (e.g. but not limited to Maxwellian)
- Continuity, momentum, Infinite hierarchy in principle. Need to achieve closure
- Collisional, collisionless, anisotropic, MHD, extended MHD, partially ionised ...

Summary

Quasineutral?
(when in a quasi-equilibrium state)



No: *these are not the
plasmas you're looking for.
See Davidson 2001!*

Yes



Fields on particles only



Lorentz force
***Response to
Background fields:***
*Guiding centre may be
useful*

Response to waves:
*wave particle
interactions and
Fokker-Planck diffusion
theory may be
appropriate*

Self-consistent fields, waves and particle



Kinetic scales



Collisional
*Boltzmann -
Maxwell*



Collisionless
*Vlasov-
Maxwell*



Fluid scales



*Thermal,
isotropic,
neutral
MHD*



*Other fluid theories
exist! E.g. **multi-fluid,**
higher-order moment,
partially ionised ...
**Some of these try to
incorporate kinetic
physics***

Particles on field only



Waves!
Which ones are possible and
grow/damp?
Consult the dielectric zoo!

- Kinetic and fluid waves
- From plasma/langmuir oscillations up to MHD scales

**(You don't always have to
start from the dielectric!) ☺**

Tips

Cgs and SI – look out for that, c and 4π factors etc

Don't be tribal – make friends with people who do other things. They may end up being your colleague/boss!

Plasmas are hard but that makes them fun - **The hardest part of classical physics!?**
Don't tell anyone I said that.

Plasma physics is all about finding the **right tool for the right job** and justifying to anyone that asks why that is ok

Other Summer Schools are available and (also!) awesome. Sometimes with low-hanging scholarships or even stipends £££\$\$\$

“ISSS”
International School for Space Simulations

Culham
Culham Plasma Physics Summer School

Les Houches Plasma Physics School

Los Alamos Space Weather Summer School

References

Chen, Introduction to Plasma Physics and Controlled Fusion, **famously pedagogical**

B&T: Baumjohann and Treumann, Basic Space Plasma Physics, **comprehensive and approachable**

T&B: more advanced topics

Krall & Trivelpiece, Principles of Plasma Physics, **Classic (my fave)**

Stix, Waves in Plasmas. **Seminal. Heavy. Definitive**

Fitzpatrick, **accessible (intellectually and practically)**
<https://farside.ph.utexas.edu/teaching/plasma/Plasma/>

Freidberg, Plasma physics & fusion energy

Freidberg, Ideal MHD

Schindler, Physics of Space Plasma Activity

Griffiths, Introduction to electrodynamics

Jackson, Classical Electrodynamics

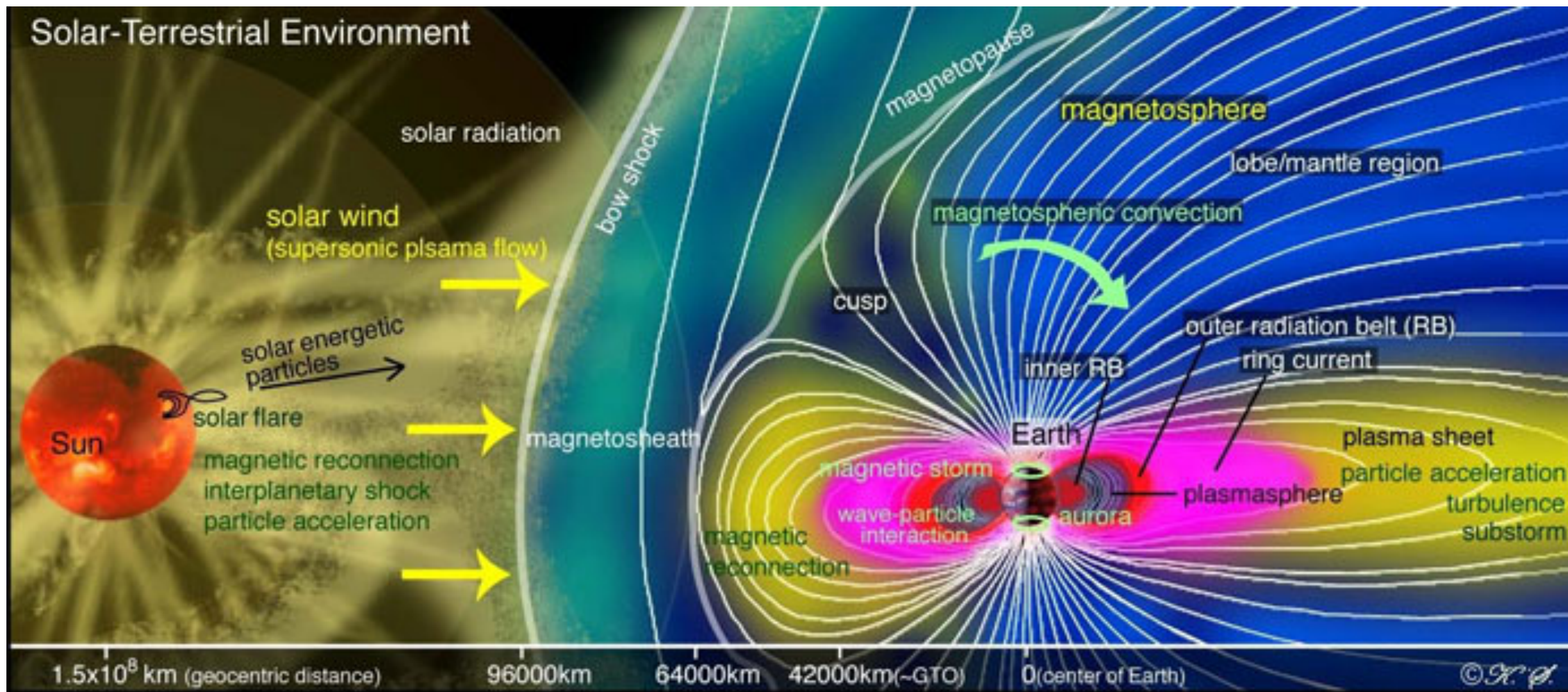
Davidson, Physics of Nonneutral Plasmas

End – extra slides

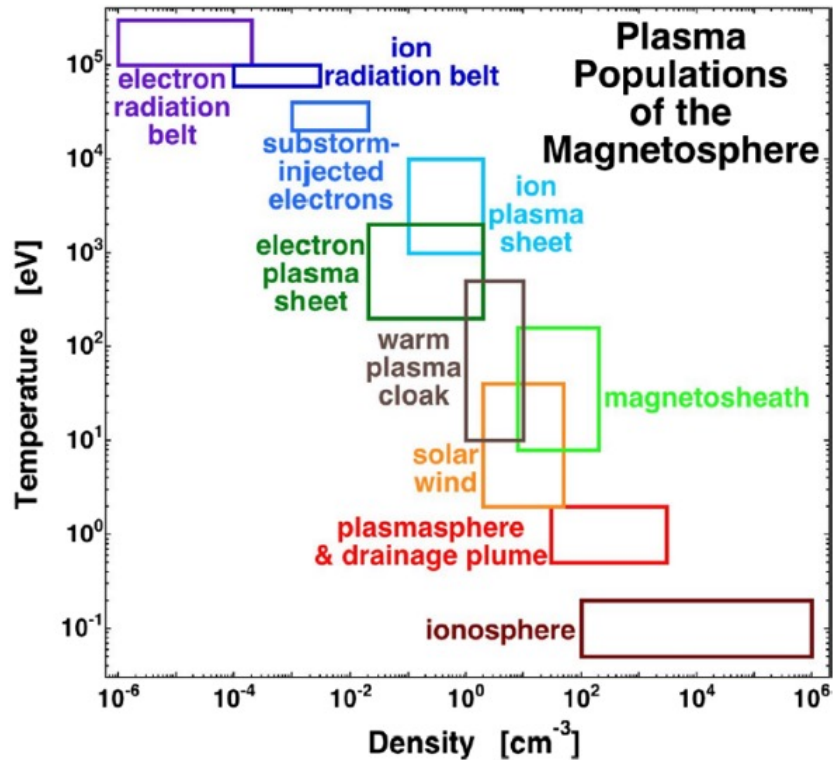
Takeaways

- Space plasmas are hot ($1\text{eV}+ \sim 11,000\text{K}+$) even when they are “cold”
- Despite being dominated by the motion of charged particles, plasmas are most usually defined by quasineutrality
 - Plasmas are often collisionless and so temperature may be a troubling concept – Use kinetic theories
- But if there are sufficient collisions (or other processes), then plasmas are often locally Maxwellian – Use fluid theories
 - If you need to look at very fast and/or small-scale process, then use kinetic theories
 - If you can look at large scale and/or long timescale processes then use fluid theories
- There is no one method that is always best. It could be single particle, kinetic, fluid, MHD, extended MHD, test-particle etc
- Plasma physics is all about finding the right tool for the right job and justifying to anyone that asks why that is ok

The solar-terrestrial system



The world of plasma physics



Borovsky & Valdivia 2018 Surveys in Geophys.
<https://msolss.github.io/MagSeminars/>

