Magnetic Reconnection
Part 1

Clare E Parnell

University of St Andrews

cep@st-andrews.ac.uk

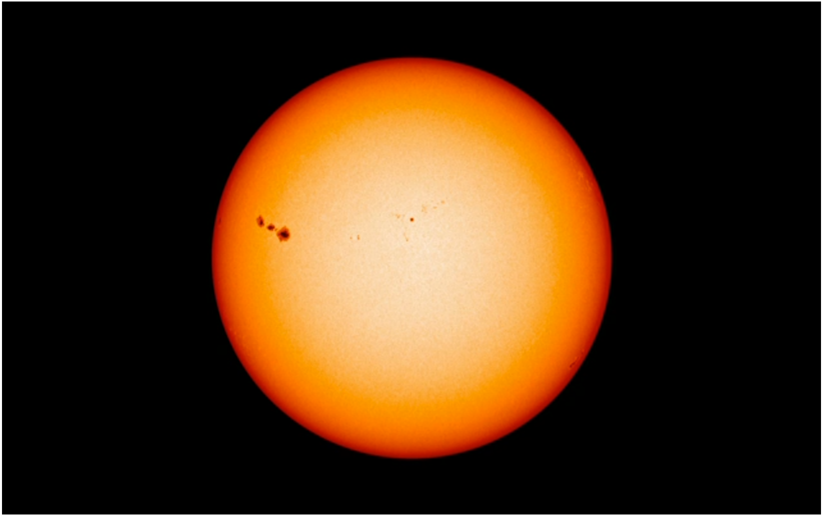
22nd August 2023

Solar & Magnetospheric Theory Group
University of St Andrews

University of St Andrews

Outline

The Sun: changes in space and temperature



The Sun: changes in space and temperature

(solar slices movie)

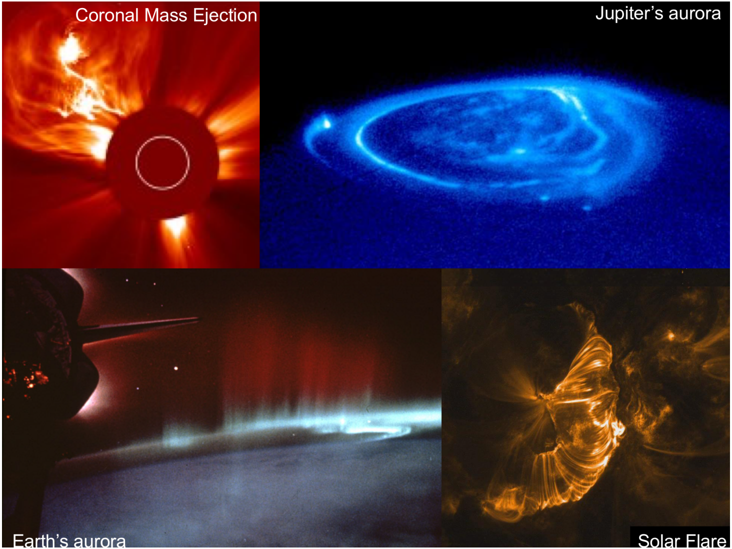
The Sun: changes in time

(swap movie)

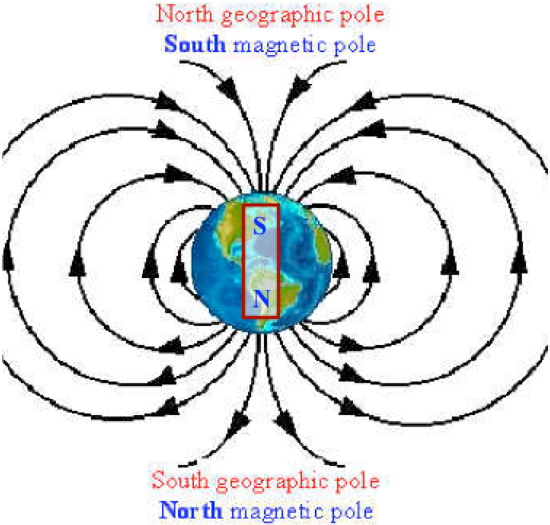
Solar Flare

(flare movie)

Dynamic Magnetic Events

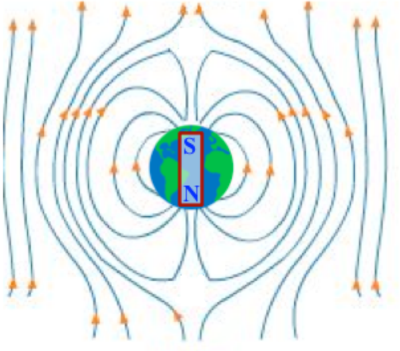


Earth's dipolar magnetic field

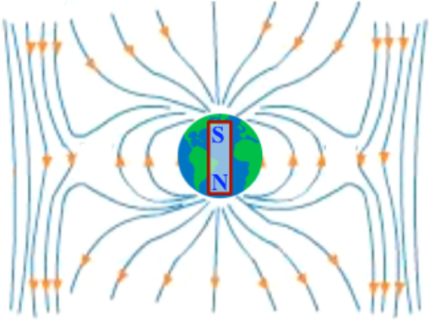


Earth's magnetic field

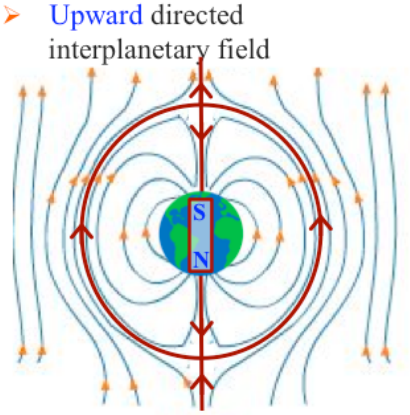
- **Upward** directed interplanetary field

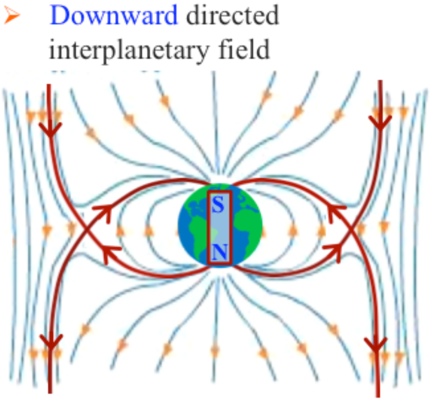


- **Downward** directed interplanetary field

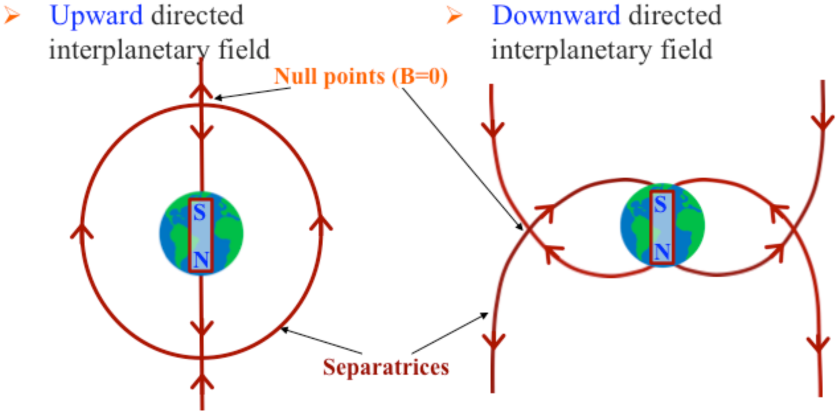


Earth's magnetic field

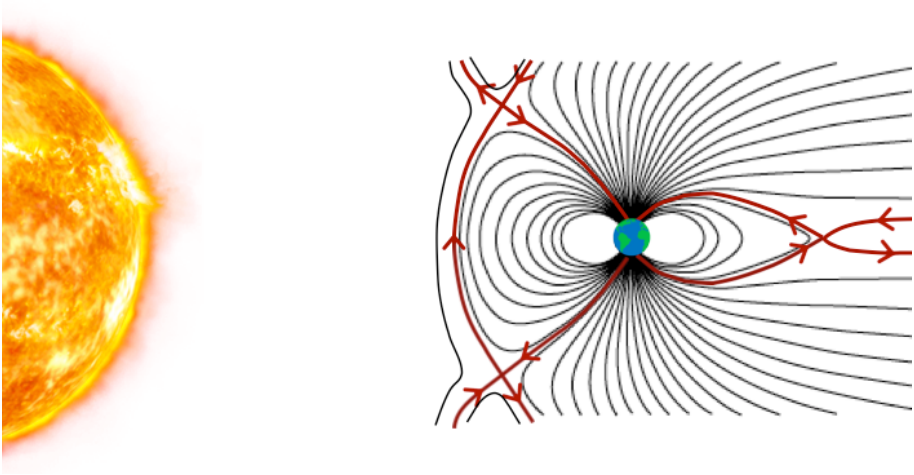




Earth's magnetic skeleton



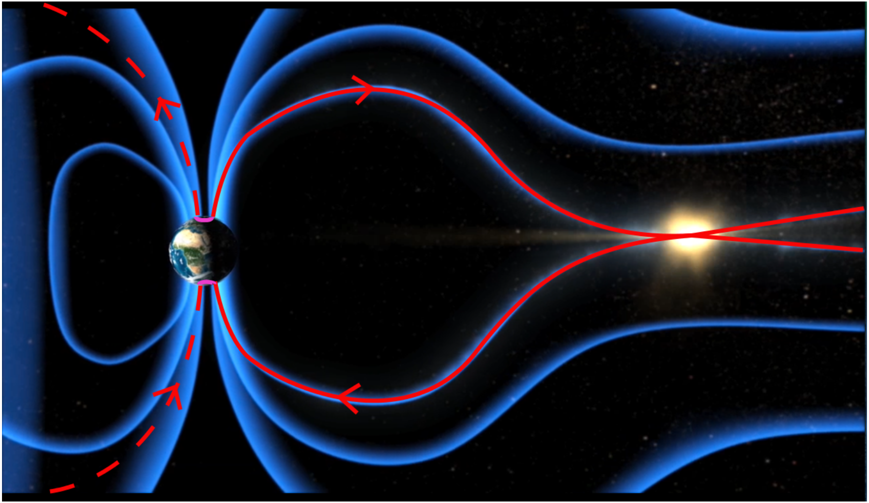
Earth's magnetic skeleton: Sun-Earth line view in 2D



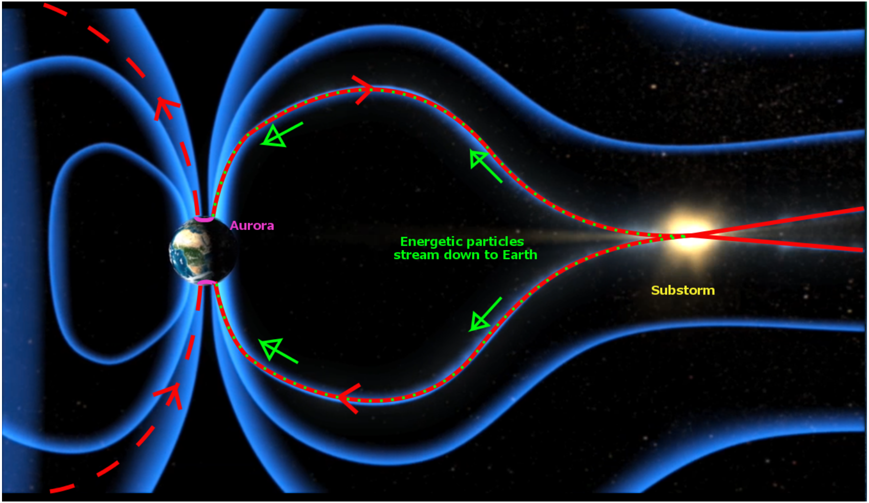
Solar storms and aurora

(cmd substorm aurora)

Solar storms and aurora



Solar storms and aurora



Equation governing changes of magnetic field

Faraday's law: $\partial \mathbf{B}/\partial t = -\nabla \times \mathbf{E}$

($\mathbf{B}$ - magnetic field, $\mathbf{E}$ - electric field and t - time)

Substitute in Ohm's law: $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{N}$

($\mathbf{v}$ - velocity and $\mathbf{N}$ - all non-ideal terms)

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{v} \times \mathbf{B})}_{\text{advection term}} - \underbrace{\nabla \times \mathbf{N}}_{\text{diffusion term}}.$$

Classical Ohm's Law

$$\mathbf{N} = \frac{\mathbf{j}}{\sigma} = \frac{1}{\sigma \mu} \nabla \times \mathbf{B}$$

($\mathbf{j}$ - electric current density, σ - conductivity, μ - magnetic permeability)

Generalised Ohm's Law

$$\mathbf{N} = \frac{\mathbf{j}}{\sigma} + \frac{1}{e n_e} \mathbf{j} \times \mathbf{B} - \frac{1}{e n_e} \nabla \mathbf{P}_e + \frac{m_e}{e^2 n_e} \frac{\partial \mathbf{j}}{\partial t}$$

Induction Equation

Induction Equation (from classical Ohm's law)

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{v} \times \mathbf{B})}_{\text{advection term}} + \underbrace{\eta \nabla^2 \mathbf{B}}_{\text{diffusion term}} , \qquad \left(R_m = \frac{lv}{\eta} \right)$$

$\eta = 1/\sigma\mu$ - magnetic diffusivity, R_m - magnetic Reynolds number.

- $R_m \gg 1 \Rightarrow$ advection dominates (most of Universe, e.g., $R_m \sim 10^{10}$ in the global corona). i.e., plasma and magnetic field move together.
- $R_m \ll 1 \Rightarrow$ diffusion dominates , i.e. magnetic field can slip through plasma.
- $R_m \approx 1$ reconnection can occur.

$$\boxed{0 < R_m \simeq 1 \text{ occurs in localised regions}}$$

Ideal regions

Ideal Ohm's Law ($R_m \gg 1$)

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{0} \qquad \Rightarrow \qquad \frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B}).$$

- Plasma ($\mathbf{v}$) and fieldline ($\mathbf{w}$) velocities perpendicular to magnetic field are equal

  $\Rightarrow$ Alfvén's frozen-in-flux theorem
- Fieldline velocity $\mathbf{w}$ satisfies $\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{w} \times \mathbf{B})$

Perpendicular fieldline and plasma velocity

$$\mathbf{w}_\perp = \mathbf{v}_\perp = \frac{\mathbf{E} \times \mathbf{B}}{B^2}.$$

"On the nature of three-dimensional magnetic reconnection" Hornig and Priest (2003)

Non-ideal (diffusion) regions

Non-Ideal Ohm's Law ($R_m \leq 1$)

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{N} \quad \Rightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times \mathbf{N}.$$

- Plasma ($\mathbf{v}$) and fieldline ($\mathbf{w}$) velocities are not equivalent.

Perpendicular fieldline and plasma velocity

$$\mathbf{w}_\perp = \mathbf{v}_\perp + \frac{(\mathbf{N} + \nabla \Psi) \times \mathbf{B}}{B^2} \qquad \Psi \rightarrow \text{scalar potential}$$

"On the nature of three-dimensional magnetic reconnection" Hornig and Priest (2003)

Proof that $\mathbf{w} \neq \mathbf{v}$ in diffusion regions

- Fieldline velocity satisfies: $\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{w} \times \mathbf{B})$.
- Faraday's law: $\frac{\partial \mathbf{B}}{\partial t} = -\boldsymbol{\nabla} \times \mathbf{E}$

$$\Rightarrow \qquad \mathbf{E} = \underbrace{-\mathbf{w} \times \mathbf{B}}_{\text{component } \perp \text{ to } \mathbf{B}} - \underbrace{\boldsymbol{\nabla}\Psi.}_{\text{'integration constant'}}$$

- Comparing with the non-ideal Ohm's Law: $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{N}$.

$$\Rightarrow \qquad (\mathbf{v} - \mathbf{w}) \times \mathbf{B} = \mathbf{N} + \boldsymbol{\nabla}\Psi$$

$$\Rightarrow \qquad \mathbf{w}_\perp = \mathbf{v}_\perp + \frac{(\mathbf{N} + \nabla\Psi) \times \mathbf{B}}{B^2}.$$

- Fieldline velocity $\mathbf{w}$ not defined at a null point $(\mathbf{B} = \mathbf{0})$
  $\Rightarrow$ the fieldlines have a discontinuous jump as they pass through a null point.

Fieldline and plasma velocities

Example: Diffusion in a current sheet

- Non-ideal Ohm's Law:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{j}/\sigma.$$

- $$\mathbf{B} = B_0 \mathrm{erf}\left(\frac{x}{(4\eta t)^{1/2}}\right)\, \hat{\mathbf{z}}.$$

- $$\mathbf{j} = -\frac{B_0}{\mu\sqrt{\pi \eta t}} e^{-x^2/4\eta t}\, \hat{\mathbf{y}}.$$

- Let plasma velocity $\mathbf{v} = \mathbf{0}$, for simplicity.

Fieldline velocity $\mathbf{w}$:

$$\begin{aligned} \mathbf{w}_\perp &= \mathbf{v}_\perp + \frac{\mathbf{j} \times \mathbf{B}}{\sigma B^2} = \frac{j_y}{\sigma B_z}\, \hat{\mathbf{x}} \\ &= -\sqrt{\frac{\eta}{\pi t}} \frac{e^{-x^2/4\eta t}}{\mathrm{erf}\left(\frac{x}{(4\eta t)^{1/2}}\right)}\, \hat{\mathbf{x}}. \end{aligned}$$

(flux velocity movie)

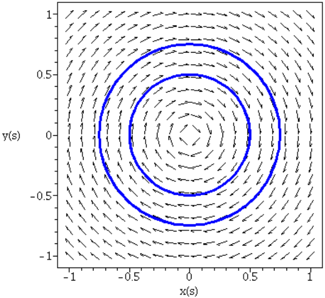
2D magnetic reconnection

- 2D reconnection occurs at magnetic null points

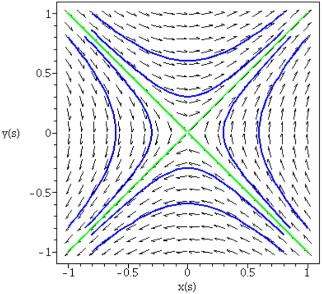
Simple 2D magnetic null model

$$B_x = B_o y / r_o \ , \qquad B_y = B_o \alpha^2 x / r_o$$

- $\alpha^2 < 0$ elliptical $\rightarrow$ O-point



- $\alpha^2 > 0$ hyperbolic $\rightarrow$ X-point
  - Limiting fieldlines $y = \pm \alpha x$ called separatrices

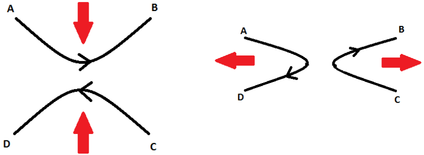


2D magnetic reconnection: main characteristics

- 2D reconnection occurs only at X-points
- Stagnation type flow required
- Reconnection requires a current sheet

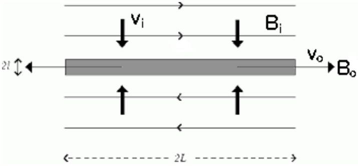
(2D reconnection movie)

- Separatrices split field up into 4 domains
- Incoming fieldlines from opposite domains break and reconnect in pairs at X-point
- Flux transferred from one pair of flux domains into another pair



Sweet-Parker reconnection "model"

- Based on the ideas of Sweet (1958) and order-of-magnitude calculations of Parker (1957).
- Considered the order of magnitude steady-state, incompressible, localised behaviour about a 2D diffusion region.
- Assume a thin current layer lies along the x-axis of length $2L$ and width $2l$ ($l \ll L$)
- Inflow into diffusion region: Steady flow and magnetic field: v_i and B_i
- Outflow from diffusion region: Steady flow and magnetic field: v_o and B_o
- Incompressible implies density constant $\rho_i = \rho_o = \rho$



Sketch of Sweet-Parker diffusion region and surrounding field.

Sweet-Parker reconnection "model": Reconnection rate

- Reconnection rate (2D):

Rate of change of flux $= \partial A/\partial t = |E|$ (from Faraday's law).

$$|E| = |vB| = M_A|B||v_A|,$$

where $M_A = v/v_A$ (Alfvén Mach number) and $\mathbf{B}(x,y) = \boldsymbol{\nabla} \times A(x,y)\hat{\mathbf{z}}$.

M_A – dimension-less measure of the reconnection rate in 2D.

Sweet-Parker reconnection rate:

$$v_i = \sqrt{\frac{\eta v_o}{L}} = \sqrt{\frac{\eta v_{Ai}}{L}},$$

since, $v_o = v_{Ai}$.

$$\begin{aligned} M_{Ai} &= \frac{v_i}{v_{Ai}} = \sqrt{\frac{\eta v_{Ai}}{L}}/v_{Ai}, \\ &= \sqrt{\frac{\eta}{L v_{Ai}}} = \frac{1}{\sqrt{R_{mi}}}. \end{aligned} \quad (1)$$

If $L = 10^7$ m – global length-scale, then $R_{mi} = 10^{12}$

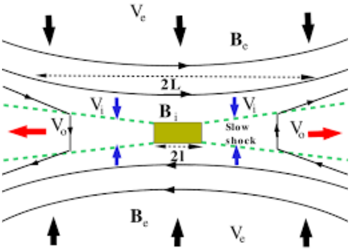
$\Rightarrow$ Sweet-parker reconnection rate:

$$M_{Ai} = 10^{-6}$$

Too slow to explain solar flares

Petschek reconnection model

- Petschek (1964) realised that a fast outflow would cause shocks to form, that could be used to speed up the reconnection rate.
- Thin current layer along x-axis of length $2L$ and width $2l$ embedded large external field $2L_e$ ($l \ll L \ll L_e$).
- External inflow region: v_e and B_e
- Local inflow region: v_i and B_i
- Outflow region: v_o and B_o
- Incompressible: $\rho_e = \rho_i = \rho_o = \rho$
- Standing slow magnetoacoustic shocks extending from corners of the diffusion region.



Sketch of Petschek reconnection model.

Petschek reconnection model

- Petschek reconnection rate:

$$\max M_{Ae} = \frac{\pi}{8 \log R_{me}}. \qquad (2)$$

If $L = 10^7$ m – global length-scale, $R_{mi} = 10^{12}$
$\Rightarrow$ Petschek reconnection rate:

$$M_{Ae} = 10^{-2}$$

Fast enough to explain solar flares.

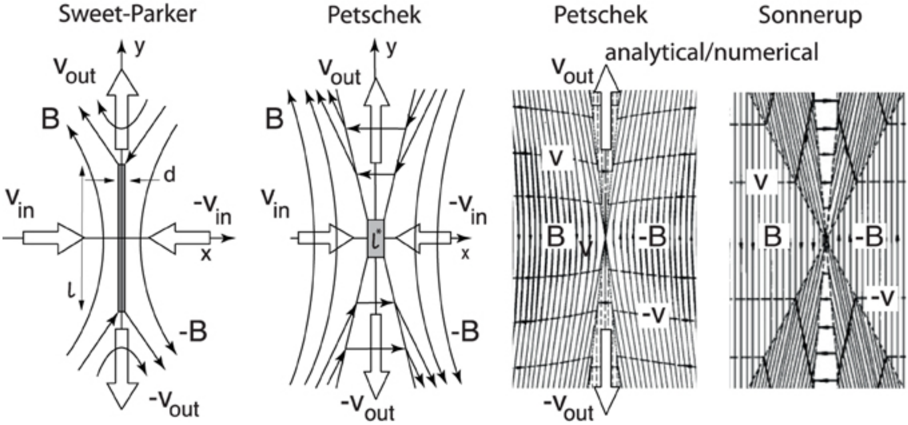
- Dimensions of current layer (diffusion region):

$$\frac{l}{L_e} = \frac{1}{R_{me} M_{Ae}}, \quad \frac{L}{L_e} = \frac{1}{R_{me} M_{Ae}^2}.$$

As reconnection rate M_{Ae} increases, the dimensions of the current layer decrease.

Other 2D steady-sate reconnection models

- Wide range of 2D steady-state reconnection models



Sweet-Parker, Petschek and Sonnerup reconnection model scenarios.

Numerical 2D reconnection “model”

(tearing-mode inst movie)

Figure: Tearing-mode Instability: a resistive (non-ideal) instability

Tearing-mode Instability:

- arises when current sheet gets too long.
- Is a 2D resistive (non-ideal) instability
- Initiates reconnection in 2D
- Reconnection then occurs at multiple 2D X-points

Magnetic Reconnection

Magnetic reconnection:

- Is a local process with significant global consequences
- Is a change in magnetic connectivity of plasma elements in a region of non-idealness
- Fundamental process of energy release $\Rightarrow$ magnetic energy converted to 1 thermal energy, 2 bulk plasma motions and 3 particle acceleration.
- Magnetic reconnection permits the restructuring of the magnetic field
- First proposed in 1940's, studied in detail since late 1950's (see, for example, Forbes and Priest 2000)

Numerical reconnection "model"

The Universe is

3D

Are there any differences between 2D and 3D reconnection?